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PROGRAMMED INSTRUCTION AS A MEANS OF TEACHING BLIND CHILDREN ADDITION AND SUBTRACTION ON THE ABACUS

Suella McCrimmon, Ed.S.
George Peabody College for Teachers
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This study was concerned with the development and evaluation of programmed instruction as a means of teaching blind children addition and subtraction on the abacus. A series of lessons was written using the principles of programmed instruction. The initial draft of those lessons was revised based on the criticism of four persons familiar with the content of the instructional program and/or the educational needs of blind children. The abacus program was presented to children with three expectations: (1) programmed instruction would be an effective means of teaching addition and subtraction on the abacus; (2) the Ss would use a given technique for bead manipulation; (3) the Ss would respond favorably to programmed abacus instruction.

The Ss were legally blind children enrolled in the public schools of Birmingham, Alabama. The age (8-18), grade level (3-11), and IQ (54-111) of the 13 Ss who began the program covered a very wide range. The program was read by the programmer to the Ss individually or to two Ss simultaneously. The children responded by completing an operation on the abacus, by silently raising a hand or displaying a

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given number of fingers, by pointing to a card which said either "Yes" or "No," or by marking with a pencil their choice of three possible answers. For every lesson each S received a frame performance score and a criterion score.

The eight Ss who completed the program were in the fifth grade or above and were, as a group, of normal intelligence and near or above grade level in all areas of mathematics achievement. They learned to apply the rules of abacus calculation with a relatively high degree of accuracy when adding and subtracting with numbers of up to four digits. The mean frame accuracy rate and the mean criterion accuracy rate of every S who completed the program fell within the 95% confidence interval established for the frame accuracy rate and for the criterion accuracy rate. Those frames and lesson sections on which performance fell below an acceptable level were revised.

Three of the Ss who completed the program developed the proposed bead manipulation procedure at an acceptable level. Since the technique presented in the abacus program was designed with reference to the needs of the tactually oriented abacist, it should be noted that the bead manipulation performance of both braille reading Ss who operated the abacus tactually was acceptable. The Ss responded favorably to programmed abacus instruction.

This study demonstrated that for some blind children programmed instruction is an effective means of teaching basic abacus skills. The program contains a large number of calculation items written with specific reference to the principles of abacus calculation. The program was of limited value in teaching a technique for bead manipulation, but additional research possibilities were suggested. A teacher might use programmed instruction as a means of teaching basic abacus skills provided that teacher controlled the rate of presentation.

PROGRAMMED INSTRUCTION AS A MEANS OF TEACHING BLIND CHILDREN ADDITION AND SUBTRACTION ON THE ABACUS

by

Suella McCrimmon

Master of Arts

George Peabody College for Teachers

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of the
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of the
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Introduction

Special education for handicapped children is based on the premise that all children should have an education which is commensurate with their abilities. The results of research conducted over a 40-year period suggest that blind children are comparable to the sighted in their ability to reason mathematically but that their ability to calculate is significantly below that of the sighted. The goal should be to provide the kind of aids and instructional materials which would eliminate that disparity.

As a consequence of his handicap, the blind student is limited in his ability to use pencil and paper, a calculation aid which sighted students take for granted. The ideal substitute should contribute to the student's understanding of number, provide concrete reliable results, be easy to handle, and be relatively inexpensive. Over time a variety of devices have been developed in an effort to meet the needs of the blind. Nicholas Saunderson, a blind mathematician who taught at Cambridge University in the seventeenth century, was one of the first to design a special calculating device. Bruce's (1963) description of 28 mathematical aids available to the blind suggests that one may be selective in choosing the aids and devices to be used in mathematics instruction.

The abacus, an ancient device almost as old as the

number system itself, has recently been adapted for and used by blind students. The recent application of the abacus to the education of the blind was reviewed in Appendix A (p. 71). Though no one method or device should be thought of as a solution to the calculation deficiency of blind students, the abacus may be the most suitable of the available aids. Franks (1970) reported a recommendation that the abacus be accepted as the basic tool for mathematical computation. Forty percent of the respondents in a survey of residential and public school programs for the blind (Lewis, 1970) considered the abacus to be of greater value to students than any other aid. Nolan and Morris (1964) and Brothers (1972) found the abacus to have a positive effect on the calculation skills of blind students.

The abacus is a deceptively simple device with which the skilled operator can perform a variety of mathematical calculations. It is possible to add, subtract, multiply, and divide with either fractions or whole numbers. The abacus may be used for calculations involving money or logarithms and to extract the roots of numbers. One must be able to add and subtract efficiently before he can develop more advanced abacus skills.

The number and arrangement of beads on the abacus is such that there is not always a one-to-one correspondence between the beads available and the number to be added or

subtracted. Therefore, addition and subtraction are frequently performed by adding or subtracting either 5 or 10 and then compensating for over-calculation by adding or subtracting numbers of lesser value. The difference between some number from 1 to 9 and either 5 or 10 is the basis for this indirect calculation. From the complementary relationship between numbers on which indirect calculation is based, it is possible to derive 17 rules which describe addition on the abacus and 17 rules which describe subtraction on the abacus. The derivation of these rules may be understood logically, but, to be an efficient abacist, one must use them as readily as he uses the multiplication tables.

Statement of the Problem

If one is to achieve ease and confidence in the use of the abacus he should have quality instruction from the beginning. A significant variable in the quality of a child's initial instruction in abacus calculation is the kind of instructional materials used. To be optimally effective such instructional materials should present the necessary information in an orderly sequence and use language appropriate to the educational level of the student for whom they are intended. The rate of presentation should be commensurate with the student's ability to absorb information and there should be ample opportunity for the student to

apply his knowledge of the new skill. For the benefit of both the student and his instructor there should be objective evaluation of the student's progress.

Published and unpublished materials about the abacus and abacus calculation were reviewed in Appendix A (p. 85). An effort was made to identify the characteristics of each that would make them of value to teachers who instruct blind children in addition and subtraction on the abacus. None of the published materials met all of the criteria described above as being characteristic of optimally effective instructional materials. Several teachers (Davidow, 1967; Fish, Forster & Ford, 1966; Jacquat, 1969; Kopecky, 1969) have written instructional materials which they felt would be more adequate than those already available to them. The lack of adequate instructional materials was recognized and discussed at the last annual meeting of the Ex-officio Trustees of the American Printing House for the Blind (103rd report, 1971). One may conclude that there is a need for instructional materials to teach blind children basic abacus skills.

Purpose of the Study

Using the principles of programmed instruction, a series of lessons were written to teach blind children basic abacus skills. That program was evaluated in a manner that would establish its effectiveness with regard to three expectations:

the adequacy of the instructional sequence, the development of a technique for bead manipulation, and the reaction of the Ss to programmed abacus instruction.

1. It was to be expected that programmed instruction would be an effective means of teaching blind children basic abacus skills, provided the programmer controlled the rate of presentation. Ss who complete the program should be able to apply the rules of abacus calculation when adding and subtracting with numbers of up to four digits. Performance data were collected so that it would be possible to determine what revisions should be made in the program.

2. It was to be expected that the Ss would use a given technique of bead manipulation. Both the Chinese and the Japanese (Crook, 1958; Dilson, 1968; Kojima, 1954; Yoshino, 1963) have concerned themselves with the most efficient use of the fingers in abacus calculation and teachers of the blind (Davidow, 1967; Kopecky, 1969; Neuman, 1970; Takeda, no date) have expressed opinions on the subject. The technique designated in the abacus program was taken from Neuman (1970). There is no known study of bead manipulation by blind students. If a given technique can be taught, perhaps other research possibilities will be suggested.

3. It was to be expected that the Ss would respond favorably to programmed abacus instruction. There was no formal evaluation procedure but behavior indicative of

either a positive or negative response to programmed abacus instruction may be noted.

Program Development

Educators of the blind have long been concerned with the development of instructional materials and devices. Caton (1969) noted that relatively little of that development was based on empirical research and suggested that a more objective approach be used. The development and evaluation of the instructional materials on which this study is based proceeded at five levels:

1. The principles of programmed instruction were used to write a series of lessons.
2. Those lessons were reviewed by four persons familiar with the content of the program and/or the instruction of blind children.
3. The lessons were revised based on the reaction of those persons who reviewed them.
4. The program was presented to children in a manner that would make it possible to monitor their performance.
5. The program was revised based on the accuracy of student response to the frames and criterion items.

Program Design

Programmed instruction is based on the assumption that a student who is exposed to appropriate experiences in the

proper sequence will develop a given proficiency. Programmed instruction is further characterized by consistent and frequent feedback to the student regarding the accuracy of his responses. A review of the pertinent literature as reported in Appendix A (p. 95) suggested that programmed instruction met the author's criteria for effective instructional materials and that programmed instruction was effective with blind children.

Mode of presentation and response. The nature of the content and the characteristics of the students for whom the abacus program was intended as well as research findings on mode of presentation and response were considered in the choice of a workable mode of presentation and response. The study by Tobin, Clarke, Lane and Pittam (1970) in which blind children responded to programmed instruction by conducting simple science experiments is comparable to programmed abacus instruction in that both require a physical response to the stimulus. In that study the instructor controlled the rate at which a taped program was read to students. One advantage of an oral presentation of the abacus program was that an oral presentation would free the students' hands to perform the bead manipulation which is associated with abacus calculation. An oral presentation would also eliminate problems associated with the relatively slow reading rate of blind children and make it possible to

present content beyond the child's reading level. The abacus program was written so that it could be presented orally.

The selection of a response mode which blind children could conveniently use also influenced the development of the abacus program. Samways, Andres, and Klaus (1964) felt that the more rapid performance of the Ss who used the aural edition of their program was a consequence of oral rather than braille responses. Coffey (1970) interpreted his findings on the response mode used by junior and senior high school students to suggest that for younger students braille may not be a thoroughly learned skill. Coffey's conclusion regarding the braille skills of junior high students would seem to be even more applicable to intermediate grade students and hence of particular significance in the development of the abacus program. The program was written so that the Ss were never required to construct braille responses. They were required to respond orally.

Instructional sequence. Programmed instruction is characterized by a carefully sequenced presentation. Therefore, the abacus lessons were written from an outline organized with reference to both the mathematical relationships and the bead manipulation associated with abacus calculation. In this way it was possible to control the level of difficulty of both the verbal instruction and the calculation items. The numbers in parenthesis in the outline

which follows are the numbers which could be added or subtracted once a S learned the rules presented in that part of the program.

- I. General orientation to the abacus.
- II. Recording, clearing, reading, and calculating directly with one- and two-digit numbers.
- III. Recording, clearing, reading, and calculating directly with three-digit numbers.
- IV. Recording, clearing, reading, and calculating directly with four-digit numbers.
- V. Addition rules
    - A. Rules that require clearing below the bar and setting above the bar (1-4).
    - B. Rule that requires clearing above the bar and setting one left (5).
    - C. Rules that require clearing below the bar and setting one left (6-9).
    - D. Rules that require clearing above and below the bar and setting one left (1-4).
    - E. Rules that require setting below the bar, clearing above the bar, and setting one left (6-9).
- VI. Subtraction rules
    - A. Rules that require setting below the bar and clearing above the bar (1-4).
    - B. Rule that requires setting above the bar and clearing one left (5).
    - C. Rules that require setting below the bar and clearing one left (6-9).
    - D. Rules that require setting above and below the bar and clearing one left (1-4).

E. Rules that require clearing below the bar, setting above the bar, and clearing one left (6-9).

Bead manipulation technique. The technique for bead manipulation presented in the abacus program was chosen after a review of ideas on the subject as reported in Appendix A (p. 85). The technique chosen was selected with reference to the needs of the tactual operator. The program specified that the Ss set the lower beads with their right thumb and that for all other bead movements they use their right index finger. The Ss were instructed to work from left to right and bottom to top in setting numbers on the abacus. They were also instructed to work from left to right and bottom to top in clearing numbers from the abacus.

Initial Review

The introductory portion of the abacus program dealt with the physical characteristics of the abacus, presented the terminology of abacus calculation, specified a technique for bead manipulation, and described the procedure for recording numbers as well as the process of addition and subtraction by direct bead manipulation. Because these lessons established the pattern of instruction for later lessons and taught concepts and procedures which were vital to an understanding of succeeding lessons, three persons with considerable experience in teaching abacus calculation

reviewed the introductory lessons and the initial presentation of addition by indirect bead manipulation. The three were provided with a taped copy of the lessons and a questionnaire. The questionnaire has been included in Appendix B.

The entire program was reviewed by an educator with broad experience as a teacher of blind children. She provided comments regarding the clarity, consistency, redundancy, and ambiguity of verbal instruction and checked all the calculation items for accuracy.

Revisions. As a consequence of the initial review several significant changes were made:

1. The program was revised to allow for presentation by the programmer rather than on tape.

2. Provision was made for the programmer to monitor the Ss' performance rather than requiring them to write their responses.

3. The revised program contained two fewer lessons and 147 fewer frames than the original.

4. The criterion items which were a part of each lesson were presented at strategic points within each lesson rather than at the end of each lesson.

5. The terms "upper" and "lower" were used to describe the relationship of beads to the bar rather than the Oriental terms "heaven" and "earth." The procedures for indirect calculation were referred to as "rules" rather than as "secrets."

Revised program. The revised program contained a total of 45 lessons. Most of the lessons were divided into two or three sections. Five variables were considered in controlling the difficulty of items within each section and of each lesson as a whole:

1. The number of digits in a numeral.

2. Whether a rule of calculation should be applied in the ones, tens, hundreds, or thousands column.

3. The number of conditions under which any one rule of calculation is applicable (varies from one to eight).

4. Whether the calculation involved two numbers or a series of numbers.

5. The number of different rules which should be applied in a single problem.

Each lesson contained from 12 to 24 frames with the mean number of frames per lesson being 19.37. Each lesson also contained a number of criterion items in which the student was expected to apply the skills taught in the instructional sequence. With the exception of Lessons One through Five, Seven, Nine, and Fifteen, each of which provided introductory explanation, all of the lessons contained a total of 15 criterion problems. The total number of criterion items in the special cases mentioned above varied with the exact nature of the content. The criterion items within each lesson were

grouped under the heading Criterion Check and distributed so that there was a criterion measure associated with each section of each lesson. Data decribing the lessons are included in Table 12 of Appendix C.

Method

Subjects

The Ss were 13 legally blind children enrolled in one elementary school and one high school in Birmingham, Alabama. They were, with one exception, enrolled in a regular school program and all of them were served by either a resource or itinerant teacher of the visually handicapped. Five of the Ss had had instruction in addition and subtraction on the abacus two years ago from a community volunteer but had not used the abacus since that time. The remainder of the Ss had never had any abacus instruction. The Ss are described in Table 1. Five of them were braille readers. The others used large print as their primary reading medium. The age (8-18), IQ (54-111), and grade level (3-11) of the Ss covered a wide range.

Procedure

Presentation of the program. The program was read by the programmer to the Ss individually or to two Ss simultaneously. Ss followed the program in pairs if:

1. They were not more than one grade apart in school.

Table 1

Subject Characteristics

S	Reading Medium	Age	IQ	Grade
1	Large Print	10	111	4
2	Large Print	11	107	5
3[a]	Braille	10	109	6
4	Large Print	10	102	5
5	Large Print	9	92	3
6	Braille	8	86	3
7[a]	Braille	11	108	5
8[a]	Large Print	12	89	6
9[a]	Large Print	11	108	6
10	Braille	14	92	9
11	Braille	18	80	11
12	Large Print	12	54	7
13[a]	Large Print	16	90	8

[a] Ss 3, 7, 8, 9, and 13 had abacus instruction two years ago.

2. Their class schedule made it possible for them to report at the same time each day.

3. Neither S was absent or tardy to a degree that would make it impossible for them to remain at the same point in the program.

4. The two Ss were able to work together without misbehaving.

5. The rate and quality of each S's response was such that neither hampered the performance of the other.

The program was presented over a 10-week period. An effort was made to present one complete lesson at each sitting. When the pressure of time made it impossible to apply this standard the break usually occurred between the frames and Criterion Check within a section or following the Criterion Check.

Response to the program. A characteristic of programmed abacus instruction was that the Ss responded at almost every step in the instructional sequence. Ss responded to the frames by: completing an operation on the abacus, silently raising a hand or displaying a given number of fingers, pointing to a card which said either "Yes" or "No," or marking with a pencil their choice of three possible answers.

The Ss also responded orally. Approximately 6% of the frames in the program were frames which, by the nature of their content or structure, required an oral response. In addition, the Ss were frequently required to respond to calculation frames by reading their results aloud. The Ss were asked to read their results aloud because even though a student understands the number value of beads he may

misinterpret his answer. The frames for which an oral response was necessary were identified before the lesson began. If more than one S were present, oral responses were elicited from each in rotation.

The Ss responded to the criterion items by: performing a designated task according to the standards established in the last frame of a lesson, setting and clearing numbers on the abacus according to a procedure taught in the program, reading numbers which were recorded on the abacus by the programmer, or calculating on the abacus. The Criterion Checks in which the Ss were required to read numbers which were recorded on the abacus were administered individually. With the exception of Lessons One through Five, Seven, Nine, and Fifteen, the Criterion Checks, which were a part of each lesson, consisted of 15 problems designed so that the Ss were required to use all they had been taught about abacus calculation. One S in each pair was asked to read his results aloud.

Immediately after they responded to an item the Ss were informed whether or not they had responded correctly. The Ss knew they had responded to the frames correctly because: they moved to the next frame without comment; the response called for in a given frame was affirmed in the content of that frame; the response called for in a given frame was affirmed in the content of the succeeding frame; or the

the programmer told the S what his response should have been.

Each time a S performed the designated criterion task accurately the programmer offered an affirmitive word or statement such as, "Fine," "Good," or "I agree." If a S responded incorrectly the programmer said, "I disagree. Your answer should be ___." If the S read incorrectly the number that was recorded on his abacus the programmer said, "Read it again." If on the third attempt a S failed to read his result correctly the programmer said, "That answer is ___." The programmer was responsible for maintaining a record of the accuracy of every S's response to each frame and criterion item.

Evaluation

Performance data from the trial of the abacus program were collected and reviewed in a manner that would establish the value of programmed abacus instruction with reference to three concerns: the adequacy of the instructional sequence, the development of a technique for bead manipulation, and the reaction of students to programmed abacus instruction. Data were collected so that it would be possible to report the amount of time required to complete the program and the accuracy with which the Ss were able to read their results.

Criteria for acceptance of lessons. Data from the performance of those Ss who completed the program were analyzed to determine the quality of each lesson within the program.

A lesson was considered acceptable if each frame and each section within that lesson met the criteria of acceptability.

A frame was considered acceptable if 75% of the Ss who completed the program were able to respond correctly. Each frame within the program was classified according to the nature of its content. The categories were: Calculation, Verbal Development, Calculation Sequence, Multiple-Choice--Rule Application, Rule Statement, Multiple-Choice--Review, and No Response. Those frames in which the Ss were required to add or subtract two-, three-, or four-digit numbers were called Calculation frames. In no Calculation frame were the Ss expected to handle more than three numbers in a series, but they were required to use one, two, three, or four rules of abacus calculation to complete the necessary computation. The frames in the category Verbal Development were designed to provide an explanation of the task at hand. Those frames in which the Ss were expected to add a long series of one- and two-digit numbers were classified under the heading Calculation Sequence. Each Calculation Sequence was designed to provide review of those rules which had already been taught. In the Multiple-Choice--Rule Application frames the Ss were required to make judgments about the application of a rule of calculation. The term Rule Statement referred to those frames to which the Ss responded by repeating a rule of calculation. In some frames a

multiple-choice format was used for review and in others no response was required.

Almost every lesson was divided into sections which consisted of a series of related frames and a Criterion Check. The relationship between the frame and Criterion Check within each section was such that several outcomes were possible.

1. If 75% of the Ss were able to complete 75% of the frames and 67% of the criterion items correctly, that section was considered acceptable. The assumption was made that performance on one was directly related to performance on the other.

2. If 75% of the Ss were able to complete 67% of the criterion items but unable to complete 75% of the instructional sequence correctly, the assumption was made that the instructional sequence was more complex than the Criterion Check and should be simplified.

3. If 75% of the Ss were able to respond accurately to 75% of the frames but unable to complete 67% of the criterion items, the assumption was made that the frames did not teach the necessary skills and should be revised.

4. If 75% of the Ss were able to complete neither 75% of the frames nor 67% of the criterion items, the assumption was made that either the frames, the criterion items, or both should be revised.

Observation of bead manipulation technique. Data collection in this area was based on the programmer's systematic observation of individual Ss as they responded to the program. The data were collected at three levels so that it would be possible to say whether the proposed technique had been taught, applied, or maintained.

1. Lesson Two and Lesson Three were designed to teach the technique for bead manipulation. Since the frames contained explicit instructions regarding bead manipulation technique, acceptable bead manipulation performance was inherent in correct response to the frames. Bead manipulation may be described as adequately taught if 75% of the Ss respond to 75% of the frames in Lesson Two and Lesson Three correctly.

2. Performance on the Criterion Checks in Lessons Four, Five, and Seven and the Criterion Check in Section A of Lessons Nine and Fifteen was the basis for determining whether the prescribed bead manipulation technique was adequately applied. Those lessons contained fewer prompts regarding bead manipulation technique than the preceeding ones and there was no reference to bead manipulation in the instructions describing the criterion task. The Criterion Check for those lessons was divided into two parts. In Part 1 each S was asked to record eight numerals on the abacus and was scored on his accuracy and on the method he used for setting beads to the bar. In Part 2 each S was

asked to read eight numerals which the programmer recorded on the abacus and scored for his accuracy and on the method he used for clearing beads from the bar. Bead manipulation may be described as adequately applied if 75% of the Ss use the correct technique at least 75% of the time in the Criterion Check for Lessons Four, Five, and Seven and in the Criterion Check for Section A of Lessons Nine and Fifteen.

3. Even though the number of references to bead manipulation decreased and was finally eliminated as the program progressed, the Ss were expected to maintain the designated technique. To determine whether the bead manipulation technique prescribed in earlier lessons was maintained, each S's bead manipulation technique was observed at specified intervals throughout the instructional sequence. The program was divided into nine divisions drawn from the outline used in writing the program. The numbers in parenthesis are the numbers of those lessons which fell into each category.

(1) Addition and subtraction with two-, three-, and four-digit numbers by direct bead manipulation (6,8,9).

(2) Addition rules that require clearing below the bar and setting above the bar (10-13).

(3) Addition rules that require clearing below the bar and setting one left (15-18).

(4) Addition rules that require clearing above and below the bar before setting one left (19, 21-23).

(5) Addition rules that require setting below the bar, clearing above the bar, and setting one left (24-27).

(6) Subtraction rules that require setting below the bar and clearing above the bar (28-31).

(7) Subtraction rules that require setting below the bar and clearing one left (33-36).

(8) Subtraction rules that require setting above and below the bar and clearing one left (37, 39-41).

(9) Subtraction rules that require clearing below the bar, setting above the bar, and clearing one left (42-45).

Every S was observed as he responded to the frames in one lesson from each division. A S was credited with having used the correct bead manipulation technique if he: (a) set all lower beads with his right thumb, (b) performed all other bead movements with his right index finger, (c) moved from left to right in setting numbers on the abacus, (d) recorded numbers by first setting the lower beads and then the upper beads or by setting the upper and lower beads simultaneously, (e) moved from left to right in clearing numbers from the

abacus, and (_f_) cleared first the lower and then the upper beads. Bead manipulation may be described as adequately maintained if 75% of the _S_s use the prescribed technique at least 75% of the time in two-thirds of the divisions in which bead manipulation performance is observed.

Behavioral reaction to programmed abacus instruction. There was no formal evaluation procedure but unsolicited comments and behavior indicative of the _S_s' reaction to programmed abacus instruction may be noted. A student who is interested or involved in any task will by the nature of his reaction demonstrate that he desires clarification or that he is able to anticipate program content. The _S_s may express a desire to go beyond the content of a daily abacus lesson. Students who are not _S_s in the study may become interested in the abacus or abacus calculation.

Results

Variation in Performance

Though all of the _S_s except one began the program with a partner, the variation in the rate and quality of response to the abacus program as the presentation progressed was such that no two _S_s who began the program together completed it simultaneously. Initially each _S_ completed one lesson per day. However, as the presentation progressed the _S_s completed more than one lesson per day either because they expressed an interest in doing so, because their schedule

was such that they were available more than one period per day, or because a doubling up was necessary if they were to complete the entire sequence during the designated trial period. Not every S who began the abacus program was able to complete it.

Subjects unable to complete the program. Five of the 13 Ss who began the program were unable to complete it. The data in Table 2 which describes those Ss are ranked according to the number of lessons each S completed. The mean IQ of this group was one standard deviation below normal. They were a year or more below grade level in arithmetic computation and arithmetic concepts. The achievement scores of those Ss who were tested in arithmetic applications showed them to be two years below grade level.

The performance of S_6 was satisfactory through Lesson Four. Those lessons provided a physical orientation to the abacus, taught the terminology of abacus calculation and the procedure for recording numbers as large as 49. In Lesson Five the 50 bead was introduced; therefore, having completed that lesson, the student should be able to record numbers as large as 99. S_6 did not follow the program beyond Lesson Five because, even after a second reading, she could not grasp the fact that the upper bead in the tens column stood for 50. Another third grader, S_5 also had some difficulty with Lesson Five but was able to complete it satisfactorily after a second reading.

Table 2

Subjects Unable to Complete the Program

					Mathematics Achievement[a]			
S	No. of Lessons Completed	Reading Medium	IQ	Grade	Grade Level When Tested	Computation	Concepts	Applications
6	5	Braille	86	3	2.9	3.1	2.5	
5	11	Large Print	92	3	2.9	2.0	2.6	
12	11	Large Print	54	7	7.3	5.4	4.8	5.3
11	15	Braille	80	11	8.8	6.4	5.7	4.9
1	22	Large Print	111	4	4.3	4.0	3.0	4.1
Mean			84.6		5.24	4.18	3.72	4.76[b]

[a] Performance on most current Stanford Achievement Test.

[b] Mean grade level of Ss tested in arithmetic applications 6.80.

Two of the Ss, $\underline{S}_5$ and $\underline{S}_{12}$, were able to perform satisfactorily as long as the required calculation could be carried out by direct bead manipulation. Calculation by means of indirect manipulation was introduced in Lesson Ten. $\underline{S}_5$ was able to complete Lesson Ten, but his performance on that lesson and the succeeding one suggested that he was responding to the verbal instruction in a rote manner without comprehending the exact nature of the task. Though both Lesson Ten and Lesson Eleven were read twice, $\underline{S}_{12}$ did not respond adequately to either of them. She seemed to comprehend that $4 + 1 = 5$ and set the five-bead but did not consistently clear four. On those occasions when she did clear four, it was as if she had checked the results mentally and found them lacking.

$\underline{S}_{11}$ could verbalize the rules of calculation but was unable to apply them accurately. On Lesson Fifteen he completed 71.4% of the frames and 33.3% of the criterion items correctly. A section by section analysis of his performance on that lesson indicated that his performance was acceptable on those items that did not require calculation but inadequate on items which it would be difficult to complete accurately without applying the rules of abacus calculation. The $\underline{S}$ reported to the programmer that he was calculating mentally. He worked very slowly and frequently asked to have content reread. $\underline{S}_{11}$ was highly motivated but very tense with

regard to the quality of his performance. He was dropped after Lesson Fifteen because the quality of his performance did not warrant the time involved nor the tension associated with his use of programmed abacus instruction.

$\underline{S}_1$ was able to learn the new rule taught in a given lesson but had difficulty retaining that rule and applying it in combination with the rules taught in other lessons. He repeated a number of lessons but the additional explanation and practice were of limited value to him. He was dropped after Lesson Twenty-two during the tenth week of the evaluation period when it became apparent that he was not achieving the hoped for success and that he would not be able to complete the program.

Subjects who completed the program. The reading medium, IQ, grade level and mathematics achievement of the eight *S*s who completed the program are presented in Table 3. The group was of normal intelligence. Mean scores on arithmetic achievement as compared with grade level at the time of achievement testing show that the group was very near grade level in mathematical computation and concepts and that they were seven months above grade level in arithmetic applications. Only data from those *S*s who completed the entire program were tabulated as a measure of program effectiveness.

Table 3

Subjects Who Completed the Program

				Mathematics Achievement[a]			
S	Reading Medium	IQ	Grade	Grade Level When Tested	Computation	Concepts	Application
2	Large Print	107	5	5.3	5.6	5.2	5.5
3	Braille	109	6	5.8	5.4	6.1	6.5
4	Large Print	102	5	5.3	4.6	5.9	7.2
7	Braille	108	5	4.8	6.0	5.2	6.5
8	Large Print	89	6	6.3	5.8	5.2	5.1
9	Large Print	108	6	6.3	6.2	5.9	7.4
10	Braille	92	9	8.8	8.2	8.8	7.9
13	Large Print	90	8	7.3	8.2	8.5	9.8
Mean		100.6		6.24	6.25	6.35	6.99

[a] Performance on most current Stanford Achievement Test.

Program Effectiveness

For every lesson each S received two scores--a frame performance score and a criterion score. The Ss who completed the program responded to 693 criterion items. Seven hundred and eighty-five of the frames in the program required a response. Of that number approximately 6% were frames

which required an oral response. When an oral response was required only one of the *S*s in a pair responded; therefore, there was some variation in the total number of frames to which each *S* responded. The data in Table 4 are ranked according to the quality of frame performance.

Table 4

Accuracy Rate of Subjects Who Completed the Program

	Frame Performance			Crit. Performance[a]	
S	Responses	Correct Responses	Accuracy Rate	Correct Responses	Accuracy Rate
3	773	634	82.0	561	80.9
8	762	641	84.1	598	86.2
7	768	677	88.1	631	91.0
13	785	698	88.9	593	85.5
2	772	697	90.2	614	88.6
4	772	701	90.8	658	94.9
9	776	711	91.6	664	95.8
10	784	743	94.7	662	95.5
Mean		687.8	88.8*	622.6	89.8**

[a] Every *S* responded to 693 criterion items.

*$p < .05$ 79.1 - 98.5.

**$p < .05$ 76.8 - 102.8.

For the program as a whole the number of correct frame responses ranged from 634 to 743. The mean number of correct

frame responses was 687.8 with a standard deviation of 36.2. For individual Ss the frame accuracy rate ranged from 82.0% to 94.7%. The mean frame accuracy rate was 88.8% with a standard deviation of 4.1. Using the value $p < .05$, the confidence interval for that mean was found to be 79.1%--98.5%. Every S's frame accuracy rate fell within that interval.

The number of correct criterion responses ranged from 561 to 662. The mean number of correct criterion responses was 622.6 with a standard deviation of 37.7. For those Ss who completed the program the criterion accuracy rate ranged from 80.9% to 95.5% with a mean of 89.8%. The standard deviation for mean criterion performance was 5.5. Using the value $p < .05$, the confidence interval for mean criterion performance was found to be 76.8%--102.8%. Every S's criterion accuracy rate fell within that interval.

Instructional weaknesses. Sixteen of the 45 lessons in the abacus program were acceptable. Each of the others contained either unacceptable frames or sections which were revised. A description of each lesson as well as data pertinent to its acceptability are provided in Table 12 of Appendix C. The lessons that were acceptable have been marked with an asterisk (*) as they appear in Appendix E.

Even though more than half of the unacceptable frames were calculation frames, only 8% of the 377 calculation

frames in the program were unacceptable. Fifty-six percent of the frames in which the Ss were required to state a rule of calculation were unacceptable. Almost 20% of the frames in the category Calculation Sequence were unacceptable. On the average an acceptable Calculation Sequence contained 13 numerals and 8.3 rules as compared with 17.2 numerals and 11.6 rules in an unacceptable Calculation Sequence. Data describing the unacceptable frames according to the nature of their content are presented in Table 5.

Table 5

Classification of Unacceptable Frames

Content	Program Total	Unacceptable Frames	Percentage Unacceptable
Calculation	377	32	8.4
Verbal Development	261	5	1.9
Calculation Sequence	51	10	19.6
Multiple-Choice--Rule Application	43	3	6.9
Rule Statement	16	9	56.2
Multiple-Choice--Review	37	0	0
No Response	42	0	0
Total	827	59	7.1

The quality of performance on the sections within each lesson was reviewed so that each section could be grouped according to one of the four possible outcomes. Performance on 98 of the lesson sections was acceptable. The largest category of unacceptable sections was those in which criterion performance was acceptable but frame performance was unacceptable. Data describing the acceptance of lesson sections are presented in Table 6.

Table 6

Relationship Between Frame and Criterion Performance on Lesson Sections

	Outcome	Number of Sections
(1)	Frame and Criterion Performance Acceptable	98
(2)	Criterion Performance Acceptable --Frame Performance Unacceptable	12
(3)	Frame Performance Acceptable-- Criterion Performance Unacceptable	2
(4)	Neither Frame nor Criterion Performance Acceptable	2
	Total	114

Reading errors. The Ss did not always read the results of their calculation correctly. The data in Table 7 which describes the accuracy with which the Ss read their results

are ranked according to the frequency with which reading errors occurred. The three Ss with the highest percentage of reading errors were braille readers.

Table 7

Frequency of Reading Errors

S	Reading Medium	Total Number of Readings	Number of Reading Errors	Percentage of Reading Errors
3	Braille	681	57	8.4
10	Braille	757	39	5.2
7	Braille	594	26	4.3
4	Large Print	674	26	3.8
13	Large Print	757	25	3.3
2	Large Print	667	15	2.2
8	Large Print	610	13	2.1
9	Large Print	675	8	1.1

Completion time. The time required to complete an instructional sequence must be considered in evaluating its efficiency. The data in Table 8 show that for those Ss who completed the program the mean time per lesson ranged from 21 to 28 minutes with a median of 25 minutes.

Table 8

Mean Time in Minutes per Lesson for Subjects Who Completed the Program

S	Hours of Instruction	Mean	sd
10	16.03	21.37	5.42
9	17.82	23.75	5.00
7[a]	17.78	24.66	6.72
8	18.23	24.71	6.53
4	18.90	25.20	8.11
13[b]	18.28	26.88	9.25
2	20.80	27.66	6.82
3[b]	19.95	27.83	8.24

[a] Time data for $\underline{S}_7$ is based on 44 scores.
[b] Time data for $\underline{S}_3$ and $\underline{S}_{13}$ is based on 43 scores.

The data in Table 13 of Appendix C show that the amount of time required to complete a single lesson ranged from a low of 9 minutes for Lesson One to a high of 62 minutes for Lesson Nine. The median of the median completion time for the 45 lessons was 24.5 minutes. The data suggest that variation in completion time was greater between lessons than within lessons.

Bead Manipulation

In order to evaluate bead manipulation performance, data were collected at three levels: instruction, application, and maintenance. There was considerable variation in the quality of performance at each level.

Instruction. Table 9 presents performance data for Lesson Two and Lesson Three, the lessons in which bead manipulation was taught. The criterion items associated with these lessons dealt with the child's orientation to the abacus, the appropriateness of his bead manipulation technique and the accuracy with which he was able to set the numbers from one to nine. In both Lesson Two and Lesson Three all of the Ss scored 75% or higher on both the frames and criterion items which dealt with bead manipulation.

Application. The scores for application of the prescribed bead manipulation technique covered a very wide range. These data are presented in Table 10. For Lesson Four and Lesson Five, six or more of the Ss used the correct bead manipulation procedure for both setting and clearing at least 75% of the time. For Lesson Seven, seven of the Ss used the correct setting procedure at least 87% of the time, but only two Ss used the correct clearing procedure at least 87% of the time. The discrepancy between application of the proposed procedure for setting beads to the bar and application of the proposed procedure for clearing beads

Table 9

Percentage of Acceptable Performance on Lessons in Which Bead Manipulation Was Taught

	Lesson Two			Lesson Three		
		Criterion			Criterion	
S	Frame	Orientation	Bead Manipulation	Frame	Accuracy	Bead Manipulation
2	80	100	100	100	100	100
3	90	40	100	75	91.7	75
4	100	100	100	85	100	100
7	95	100	100	95	100	100
8	75	80	100	100	100	100
9	95	100	100	100	100	100
10	90	100	100	100	100	100
13	100	100	100	100	100	100

from the bar is even more apparent when one examines the application scores for Section A of Lessons Nine and Fifteen. For these measures, seven of the Ss had scores of 75% or better for application of the setting procedure. Four Ss had scores of 75% or better for application of the clearing procedure.

Maintenance. Each S's use of the proposed manipulation procedure was observed at specific intervals throughout the

Table 10

Percentage of Acceptable Performance on Criterion Measures in Which Application of Bead Manipulation Technique Was Evaluated

							Section A			
	Lesson 4		Lesson 5		Lesson 7		Lesson 9		Lesson 15	
S	Set	Clear	Set	Clear	Set	Clear	Set	Clear	Set	Clear
2	87.5	87.5	62.5	75	100	62.5	100	87.5	100	75
3	12.5	25	12.5	62.5	12.5	25	12.5	0	0	0
4	100	25	100	75	100	25	75	12.5	100	12.5
7	100	100	87.5	100	100	100	100	100	100	87.5
8	100	100	75	87.5	87.5	50	100	100	100	87.5
9	100	100	87.5	87.5	100	37.5	100	25	100	0
10	100	100	100	100	100	87.5	87.5	100	100	100
13	100	100	100	87.5	100	37.5	100	0	100	12.5

program. Three of the Ss used the prescribed bead manipulation technique at least 75% of the time in six or more of the lessons in which their manipulation technique was observed. $\underline{S}_3$'s manipulation maintenance scores were consistently very low. Data on the maintenance of bead manipulation technique are presented in Table 11.

Behavioral Reaction to Programmed Abacus Instruction

All of the Ss who completed the program displayed one or more behaviors indicative of a positive response to programmed abacus instruction. That response took several forms: comments which reflected achievement satisfaction, anticipation of program content, and expressions of special interest in abacus calculation.

The comments and performance of more than half of the Ss who completed the program indicated that they were conscious of the developmental sequence and had some grasp of the numerical relationships on which program organization was based. The Ss were able to achieve some success with programmed abacus instruction and seemed to value that success. Three of the Ss unable to complete the program showed some satisfaction with their limited achievement. The only negative reaction evident was from two Ss who failed to complete the program.

The demands of experimental control made it necessary

Table 11

Percentage of Acceptable Performance on Program Division in Which Maintenance of Bead Manipulation Technique Was Evaluated

S	Program Division								
	1	2	3	4	5	6	7	8	9
2	54.5	64.3	63.6	43.0	46.6	57.1	61.5	72.7	46.1
3	0	6.6	0	0	0	0	0	0	0
4	63.6	41.8	18.2	40.0	7.0	25.0	21.4	45.4	58.2
7	81.8	78.6	83.3	100	100	100	92.3	100	92.3
8	90.9	76.4	33.3	52.6	93.3	100	53.8	100	90.9
9	90.9	50.0	36.3	31.2	7.0	28.5	46.1	18.2	18.1
10	90.9	92.8	85.7	100	93.3	100	92.3	81.8	83.3
13	23.1	35.7	64.2	31.2	46.6	92.8	58.3	27.2	58.2

to deny the requests of at least three Ss who asked for problems in addition to those included in the program. After evaluation of the abacus program was complete, the Ss were given additional opportunities to develop their abacus skill. One S was presented with the doubling problem in which one adds 1, 2, 4, 8, 16 . . . until the abacus is filled to capacity. His interest in that problem spread to other students in the resource room. All of the Ss who completed the program accepted the programmer's offer to explain multiplication and division on the abacus. All of the Ss who completed the program and two who did not complete the program participated in an abacus contest held after evaluation of the abacus program was complete.

Interest in the abacus program went beyond those children who were Ss in the study. Both second graders served by the resource room from which the Ss were taken learned to record numbers of two and three digits and to add and subtract when calculation required only direct bead manipulation. All of their instruction was voluntary. The students in one classroom in which a S used his abacus expressed an interest in learning abacus calculation.

Discussion and Conclusions

Those concerned with the development of programmed instruction as a teaching technique consistently emphasize

the relationship between presentation, evaluation of results, and revision. Therefore, the development of programmed instruction becomes a continuous, ongoing process characterized by an ever-increasing refinement of both the program and the evaluation procedures. The program to teach blind children addition and subtraction on the abacus was developed with three expectations: that programmed instruction would be an effective means of teaching basic abacus skills; that a given technique of bead manipulation could be developed; and that children would respond favorably to that method of instruction. Results from an evaluation of that program suggested that for some Ss programmed abacus instruction was an effective means of teaching basic abacus skills and that those Ss responded favorably to it. The program was of limited value in developing a technique for bead manipulation. Those results are discussed here as they relate to further development and evaluation of the abacus program.

Program Effectiveness

For the program as a whole every Ss frame accuracy rate fell within the confidence interval established for the mean frame accuracy rate of that group which completed the program. The lower limit of that confidence interval, 79.1%, was higher than the 75% standard used in determining the acceptability of the frames and sections within each lesson. As compared with the level of achievement

considered acceptable, the quality of overall frame performance was relatively high. The 75% standard used in this study would seem to be a minimum one. In further evaluation, one might well consider setting a higher standard of frame acceptability, especially if that study included a larger number of Ss.

The criterion accuracy rate for each of the eight Ss who completed the program also fell within the confidence interval established for that group's mean criterion accuracy rate. The standard for acceptable criterion performance used in determining section acceptability was that 75% of the Ss complete 67% of the criterion items correctly. The confidence interval for the mean criterion accuracy rate was 76.8%--102.8%. Here again the implication is that the standard set was a minimum one and that in further evaluation of the abacus program the standard of acceptability might be raised.

Even though the performance of those Ss who completed the program was of a relatively high quality, an analysis of their performance suggested that there were certain steps in the instructional sequence at which their performance fell below an acceptable level. Such points were regarded as weaknesses in the instructional sequence and revisions were made. Those revisions are described in Appendix D. The program as revised is reported in Appendix E.

Though scores representing the accuracy of their responses were not reported in detail, the performance of those Ss who were unable to complete the program was reviewed in an effort to identify the kinds of difficulty which made it impossible for them to respond to programmed abacus instruction. $\underline{S}_6$ was unable to satisfactorily complete the lesson in which the 50 bead was used to record numbers from 50-99. Her teacher reported that $\underline{S}_6$ was deficient in her understanding of place value. Two Ss, $\underline{S}_5$ and $\underline{S}_{12}$, were unable to grasp the relationship between numbers on which indirect calculation is based and were dropped at the point at which they were required to apply that notion. The performance of all three Ss was adequate up to the point at which they were dropped.

In contrast, $\underline{S}_{11}$ and $\underline{S}_1$, until they were finally dropped, responded adequately to some items and inadequately to others. $\underline{S}_{11}$ could verbalize the rules of abacus calculation but was unable to apply them accurately. Because he did not retain the rules of abacus calculations, $\underline{S}_1$ had difficulty with those problems in which he was expected to use more than one rule of calculation. In their development of programmed instruction, Sidman and Stoddard (1966) noted that for their unsuccessful Ss "Errors created more errors (p. 165)."--a phenomenon which was very apparent in the performance of both $\underline{S}_1$ and $\underline{S}_{11}$. Having assumed that an initial error may have

come from a lack of attention or from a lack of understanding, Sidman and Stoddard defined a procedure for repetition of program content based on their understanding of sources of error within a program. Before being dropped, every S who failed to complete the program was given the opportunity to repeat the lesson or lessons on which his performance was inadequate. However, the evaluation procedure did not provide criteria for lesson repetition or a procedure for evaluation of differences in performance on the first and second trial. A procedure comparable to that used by Sidman and Stoddard would have been especially useful with the Ss whose performance was of inconsistent quality. Any further evaluation of the abacus program should include provisions for repeating content and standards for dropping Ss from the program.

Relationship of subject characteristics to program effectiveness. Although the data reported here suggest that performance on the abacus program was of a relatively high quality, it should be noted that they are based only on the performance of those Ss who completed the entire program. They were in the fifth grade or above, of normal intelligence, very near grade level in arithmetic computation and concepts, and above grade level in arithmetic applications. Since they were near or above grade level in all areas of mathematics achievement, the Ss who completed the abacus program

were not typical of those mathematics students about whom educators of the blind have expressed the greatest concern. One may draw either of two suppositions: the Ss who completed the abacus program were able mathematics students and would do well with any method of instruction, or blind students who achieve at grade level without the abacus would be able to achieve at grade level or above with a comprehensive program of abacus instruction. This study was not designed to evaluate the effect of abacus instruction on mathematics achievement.

As a group those Ss who failed to complete the program were one standard deviation below the norm in intelligence and from one to two years below grade level in all areas of mathematic achievement. Two of them were children of low intelligence and three were in the third or fourth grade. However, such children are not necessarily incapable of learning abacus calculation. Several authors (Brothers, 1972; Bruce, 1970; Kohr, 1963; Nolan & Morris, 1964) reported that less able students responded well to abacus instruction. As time allowed those Ss who failed to complete the program were instructed by the programmer in an informal manner designed to meet their individual needs. All of them responded favorably to the more individualized instruction. The needs of those Ss were so varied that it would be difficult if not impossible to design a single program that would

meet every need.

Reading errors. Requiring a child to read his results aloud presents him with a task different from that presented if those results are read for him. The Ss were asked to read their results aloud primarily because the programmer felt that, even though ths Ss understood the numerical value of the beads, they might misinterpret their answers. Every S made some errors in reading his results, but the highest percentage of errors were made by braille readers. S_3, a braille reader who had "light perception" but read his results visually made almost twice as many errors as any other S. The errors may have occurred because the Ss were careless in examining their results or because they failed to see all the beads that were set to the bar. Other researchers might consider the determination of an acceptable error rate and whether the number of reading errors decreases as one's experience with the abacus increases. The Ss visual level and the physical characteristics of the abacus they use are variables which should be considered in any such study.

Completion time. The abacus program was read to the Ss over a 10-week period. For those Ss who completed the program, mean completion time ranged from 21 to 28 minutes per lesson. The median completion time was 25 minutes, somewhat less than the expected 30 minutes per lesson. At one

time or another all of the Ss completed more than one lesson per day and one S completed almost half the program in the last 2 weeks of the evaluation period. The data on program completion time suggest that, depending on the amount of time devoted to abacus instruction each day, it would be possible to complete the abacus program in somewhat less than 10 weeks.

Development of Bead Manipulation Technique

The abacus program was of limited value in developing a technique for bead mainpulation. A review of the bead manipulation performance of the Ss in this study suggests that the failure of the program in this area was related to the visual level of the Ss and that the Ss generally failed to use the specified procedure for clearing beads from the bar.

Performance at three levels of development. In Lesson Two and Lesson Three, which were designed to teach the technique of bead manipulation, all of the Ss scored 75% or higher on both the frames and criterion items which dealt with bead manipulation. The results suggest that bead manipulation was adequately taught. Since no item involved numbers of more than one digit, the left-to-right sequence in which numbers were set or cleared from the bar was not a factor in the evaluation of performance.

The Criterion Checks in which application of the bead manipulation technique was evaluated involved numbers of two, three, and four digits. Each *S* received two scores, one for setting beads to the bar and another for clearing beads from the bar. Six of the eight *S*s who completed the abacus program used the correct setting procedure at least 75% of the time in every lesson in which the application of bead manipulation was evaluated. Six of the *S*s cleared in the prescribed manner at least 75% of the time in Lesson Four and Lesson Five, but in Lesson Seven only two *S*s met that standard. In Section A of Lessons Nine and Fifteen, four *S*s used the correct clearing procedure at least 75% of the time. In short, there was considerable discrepancy between the scores for application of the bead manipulation technique in setting as compared with scores for application of the bead manipulation technique in clearing.

There are at least two factors which may have contributed to this discrepancy. The frames in those lessons in which application of bead manipulation technique was evaluated contained fewer prompts regarding bead manipulation technique, and there was no reference to bead manipulation technique in the instructions regarding the criterion task. It should also be noted that the numbers which the *S*s were asked to set and clear contained more digits than those in the earlier lessons. The increased number of digits meant that movement

from left to right became a factor in the application of the procedure for setting and clearing numbers from the abacus. Apparently the added variable did not influence the accuracy with which the specified setting procedure was applied but did have considerable effect on application of the clearing procedure.

Knowing that the Ss applied the prescribed procedure for setting but not for clearing, one should not be surprised to find that when both processes were included in the definition of acceptability the scores were unacceptable. Only three of the eight Ss maintained the prescribed technique; that is, they used it at least 75% of the time in six or more of the lessons in which their bead manipulation technique was observed. Two of the three, S_7 and S_{10}, were the only braille Ss in the study who operated the abacus tactually.

Individual differences in bead manipulation performance. The technique for bead manipulation prescribed in this study was presented under the assumption that it would facilitate abacus calculation by students who operate the abacus tactually. Therefore, the manipulation performance of the braille readers who completed the abacus program is of special interest. Two of the three Ss who performed in an acceptable manner at all three levels of development were braille readers who operated the abacus tactually. Four of

the Ss learned to use their thumb and index finger correctly and to set numbers on the abacus in the proper sequence but did not learn to clear the abacus as the program specified. One S seemed to absorb none of the instruction in bead manipulation. One may well ask why a majority of the Ss failed to develop the proposed technique for bead manipulation.

The bead manipulation scores of S_3, a braille reader with enough residual vision to operate the abacus visually, were consistently very low. S_3 placed the abacus flat on his desk with a short side parallel to his body so that the portion with the most beads was to his left. He steadied the abacus with his right hand and placed his forehead on his right hand so that his nose practically touched the abacus. He used the index finger of his left hand to move beads back and forth. S_3 appeared to be using the vision in his left eye to focus on the beads. The approach was functional; but S_3 received very little credit for his bead manipulation performance because the approach he used was not that specified in the program.

S_{13}'s bead manipulation scores were unacceptable because she cleared from top to bottom rather than from bottom to top. Her bead manipulation score for Division 6 as shown in Table 11 was considerably higher than that in any other division. That score is from a lesson in which a number

smaller than five was subtracted from five so that the results did not involve beads from above the bar. Since the variable top to bottom was eliminated, her score was unusually high. She was the only print reading S who did not set upper and lower beads to the bar simultaneously.

The bead manipulation performance of the remaining Ss did not follow a clear-cut pattern. They most often lost credit on bead manipulation because they used a procedure for clearing beads from the bar other than that specified in the program. The Ss occasionally cleared the upper beads with their right thumb. They simultaneously cleared all the upper beads and then all the lower beads or visa versa. At other times they began clearing on the right hand edge and worked to the left.

Two of the Ss who were unable to complete the program were left-handed. They made some effort to use their right hand, but as the presentation progressed, both boys began using their left hand. Although handedness was not the primary source of difficulty for either S, if they had completed the program their bead manipulation scores would have been quite low.

Conclusions regarding bead manipulation performance. The choice of a manipulation technique should foster the development of calculation rather than hinder it. The technique described in this program was presented under the

assumption that it would facilitate calculation by blind students who operate the abacus tactually. Therefore, it should be noted that both of the braille Ss in this study who operated the abacus tactually had consistently high scores on all measures of bead manipulation.

It is possible to calculate accurately on the abacus with a manipulation procedure other than the one presented in this study or without using a specific procedure. The fundamental question is not how one moves the beads but whether or not he calculates accurately. Having recommended a manipulation procedure, one should consider individual differences. Given one S's determination to use his residual vision, it must be concluded that for him the proposed method was not a good choice. Two left-handed Ss failed to complete the program. The emphasis in further instruction should be that these Ss become skilled abacists rather than that they become either right- or left-handed abacists.

For those Ss who completed the program, the primary source of bead manipulation error was their failure to use the designated procedure for clearing beads from the bar. Perhaps the partially sighted Ss failed to follow the prescribed procedure because they were able to see another way of clearing the abacus that was easier for them. One of the procedures they used may be more efficient than that

suggested by the programmer.

Reaction to Programmed Abacus Instruction

The Ss responded favorably to programmed abacus instruction, but that reaction may have been directed toward the content of the program or toward the method of instruction. Several teachers (Kohr, 1963; Lewis & Coker, 1971) reported that their students responded favorably to abacus instruction. None of the Ss are known to have had prior experience with programmed instruction; therefore the novelty of the approach may have been a factor in their positive reaction.

Implications for Education

Those children who completed the program learned to apply the rules of abacus calculation with a relatively high degree of accuracy when adding and subtracting with numbers of up to four digits. Although use of the abacus program required a live reader to monitor and reinforce responses, it did allow for some individualization in the rate of presentation. Because of differences in the rate and quality of student response, no two Ss who began the program together completed it simultaneously. One of the teachers whose students were Ss in this study learned to subtract on the abacus by following part of the program with a S. A teacher could learn abacus calculation by following the program a lesson ahead of her students.

Since the responses required from the reader are included in the program, the program might be read for students by a person with no knowledge of the abacus.

Given the feasibility of programmed instruction as a means of teaching basic abacus skills and the suggestion from other research that less able students do respond to abacus instruction, one may ask what characteristics a program should have if it is to be effective with students like those who failed to complete the program evaluated in this study. The programmer should define the level of mathematics achievement required of those students who will use the program. A primary consideration should be whether the prospective students are expected to understand place value or whether the programmer will use the abacus to teach place value. Special care should be taken in presenting the relationship between numbers which is the basis for indirect calculation. Further research may define a level of mathematics achievement prerequisite for the comprehension of indirect calculation.

In this program the Ss were frequently expected to apply more than one rule of abacus calculation in a single problem. Reinforcement was offered only after the Ss completed a problem; therefore, each S was required to think through for himself the exact source of his error. One would expect that task to be especially difficult for less able children. A program designed to compensate for that deficiency would

of necessity move in smaller steps and, therefore, more slowly than the program evaluated in this study.

Implications for Research

The development of programmed instruction is by its very nature an ongoing process. The evaluation reported here and the revisions which were a consequence of that evaluation should be regarded as the first, rather than the last, step in the developmental process. Further evaluation should be conducted to determine whether revisions in the program enhanced its effectiveness.

A primary concern of the initial presentation was the development of a technique for bead manipulation. The *S*s applied the proposed procedure for setting numbers on the abacus but did not apply the procedure for clearing. The *S*s most frequently lost credit on bead manipulation maintenance because they failed to apply the designated procedure for clearing beads from the bar. In further research one might either not teach a technique for clearing beads from the bar or teach a technique comparable to those used by the *S*s in this study. Performance data from any further evaluation of bead manipulation performance would be of greater value if the *S*s were scored separately on each manipulation criteria.

This study did not provide data on the order in which *S*s set or cleared beads from the bar when they calculated

indirectly. However, an implication of Neuman's (1970) recommendation that one begin bead manipulation in the column where he is calculating and that he move lower beads before moving the upper beads is that he add 1 by clearing 4 and setting 5 or that he add 6 by clearing 4 and setting 10. The manner in which the Ss stated the rules of abacus calculation and the sequence in which they were observed to apply the rules of calculation suggests that it may be easier for students to understand and apply the rules of calculation if they either add 1 by setting 5 and clearing 4 or add 6 by setting 10 and clearing 4. The order in which beads are set or cleared from the bar in indirect calculation is a far more important question than the sequence in which a result is cleared from the abacus.

In a majority of both the calculation frames and criterion items, the Ss were required to apply more than one rule of abacus calculation. This study was not designed to provide data on the source of difficulty within an item, but the Ss were observed to make certain kinds of errors. They tended to confuse pairs of rules which involved numbers whose sums are commutative. For example, the rule for adding 8 was confused with the rule of adding 2. The sequence of presentation was such that in some cases two such rules were presented in succession. A rule for adding 3 was taught immediately following a rule for 2, a rule for 6

immediately following a rule for 4, and a rule for 1 immediately following a rule for 9. The Ss were occasionally observed to confuse a rule of addition with a rule of subtraction. Here again, the confusion seemed to involve numbers like 3 and 7 or 8 and 2 whose sums are commutative. A study which identified more precisely the nature of errors made by beginning abacists might also provide some insight into the best way of dealing with them. Such a study might show that some rules are more difficult than others.

Assuming that programmed instruction is an effective means of teaching basic abacus skills one may ask whether it is any more or less efficient than some other method of introducing blind children to abacus calculation. The quality of performance following instruction and the time involved in providing that instruction are factors that should be considered in making judgments about the efficiency of one method as opposed to another. One might also consider the qualifications of those persons who teach basic abacus skills with programmed instruction as opposed to the qualifications of those persons who teach basic abacus skills with some other technique.

Topics for further research in abacus instruction have been suggested, but abacus instruction should not become an end in itself. The primary question is the effect of skill in abacus calculation on mathematics achievement.

The performance of those Ss in this study who completed the abacus program was of a relatively high quality. As a group, they were children of normal intelligence who scored near or above grade level in all areas of mathematics achievement. Brothers (1972) and Nolan and Morris (1964) found that skill in abacus calculation enhanced the calculation achievement of blind children. Perhaps students who already achieve at or near grade level would be able to achieve above grade level if they had a comprehensive knowledge of abacus calculation. That possibility should be considered in further research.

Summary

Using the principles of programmed instruction, a series of lessons were written to teach blind children addition and subtraction on the abacus. As a measure of their effectiveness, the lessons were read to legally blind children enrolled in a public school program for the visually handicapped. The frame accuracy rate and the criterion accuracy rate of every S who completed the program fell within the 95% confidence interval for mean frame accuracy and for mean criterion accuracy. The data suggested that the intellectual ability, grade level, and mathematics achievement of those Ss were important factors in their ability to profit from programmed abacus instruction. The program was revised based on the performance of the eight Ss who completed the program. Three

of the eight Ss developed the prescribed bead manipulation technique at an acceptable level. Since the bead manipulation procedure presented in the program was developed with reference to the needs of the tactually oriented abacist, it should be noted that both of the braille reading Ss who operated the abacus tactually had acceptable bead manipulation scores. The Ss reacted favorably to programmed instruction in addition and subtraction on the abacus. The classroom teacher might use programmed instruction as a means of teaching basic abacus skills provided that teacher controlled the rate of presentation.

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APPENDIXES

APPENDIX A

REVIEW OF THE LITERATURE

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There are several variables to be considered in the development of a program to teach blind children addition and subtraction on the abacus. A primary factor is the mathematical ability of blind children and their mathematical competence as measured by tests of achievement in calculation. A second variable is the effectiveness of the abacus in facilitating mathematical calculation by blind children. A third consideration is the availability of instructional materials for teaching abacus calculation. A variable in the type of instruction offered is the emphasis or lack of emphasis on a technique for bead manipulation. Finally, one must consider the potential of programmed instruction as a means of teaching abacus calculation.

Achievement in Mathematics

The mathematical ability of blind children and their ability to calculate has been studied over a 40-year period. Though one would expect to find a direct relationship between the two, such is not the case.

Arithmetic Reasoning

Merry (1931) used the Stevenson Arithmetic Test Form 1 as a measure of problem solving ability. Her subjects were 170 children in grades four through six from six residential schools for the blind. She found that blind children tested

at just about grade level and that they had difficulty with practically the same questions as the sighted. In a study of the educational achievement of fifth and sixth graders in 12 schools for the blind, Abel (1938) found that children in both grades made their best scores in arithmetic problems.

Hayes's review of mathematical achievement by the blind (1944) showed them to be comparable to the sighted in problem solving ability. Records for 1,619 blind children on tests of simple reasoning as measured by the Binet-Simon Intelligence Test showed almost the same percentage passing as among Terman's 3,000 Ss ages 12-18. The scores of 264 children tested at Perkins School for the Blind on the arithmetic problems section of the Weschler Adult Intelligence Scale were equal to the adult norms. Hayes reported that the scores of 600 children in nine schools for the blind on the test of arithmetic problem solving from the Stanford Achievement Test (SAT) approximated the sighted norm. He noted that children in schools for the blind were about two years over age for grade but concluded, "We may say that in the use of arithmetic in thinking quantitatively about everyday problems blind children do just about as well as seeing children in the same grades (p. 105)."

Tillman (1967) compared the performance of blind and sighted children on each of the five subtests of the Weschler Intelligence Scale for Children. His findings were based on

results from 110 Ss with a mean age of 9.9 years. Performance on the arithmetic test showed the blind to be "quite comparable to the sighted in numerical ability (p. 112)." One may conclude that the ability of blind children to reason mathematically is comparable to that of their sighted peers.

Computation

Although the problem solving ability of blind children seems to be comparable to that of sighted children, several studies have shown their achievement in computation to be somewhat less than what one would expect from children of normal ability. Hayes (1937) adapted the SAT for use with blind children and presented it in two successive years at Perkins School for the Blind and the New York Institute for the Blind. He proposed that norms for the sighted be used on the first nine tests. His conclusion regarding Test 10 (Arithmetic Computation) was that it "introduces certain difficulties and appears to demand abilities which we cannot reasonably expect from the average blind pupil, even when a liberal time allowance is granted. For the present we would suggest that in this test the seeing norm for the next lower grade be used as the standard for each grade in schools for the blind (p. 90)."

Nolan and Ashcroft (1959) reported further study by Hayes which showed retardation in the computational

achievement of blind children. Hayes analyzed the computation scores of 600 children in grades four through nine on the SAT and found them to be 20% below the norm. He suggested that the content was not suitable for blind children; hence, the computation tests from the SAT were not published by the American Printing House for the Blind until 1959.

Nolan and Ashcroft (1959) evaluated an experimental braille edition of the computation tests from the SAT by testing the performance of 282 children in nine residential schools. They administered Form J of the computation tests to children at the primary, elementary, intermediate, and advanced level. Approximately 75% of each group fell below the expected achievement for their grade. The median retardation for third graders was seven months. The degree of retardation increased with age so that eighth graders showed a retardation of one year and one month. Overall, the scores were approximately 16% below the sighted norm.

Nolan (1959) described the areas of low achievement on each level of the computation test. Low achievement existed if "fewer than 40% of the pupils taking the test answered problems of a specific type correctly (p. 128)." Results from the primary and elementary levels are reported here since the skills they measure are related to the skills taught in the abacus program. The areas of low achievement defined by the primary battery were: addition or subtraction

with carrying, addition or subtraction involving numbers of more than two digits, addition of decimals, any type of multiplication, and any type of division. The areas of low achievement described by the elementary battery were: addition of more than two numbers with more than two digits, subtraction of numbers with more than two digits, division and multiplication with more than one digit, and any operation involving decimals.

In his study of the computational achievement of 269 braille readers in 12 residential schools for the blind, Brothers (1972) used procedures similar to those of the earlier study by Nolan and Ashcroft (1959). He found that the achievement of students in all grades was consistently below that of students in the 1959 study. The median scores for each grade level ranged from four to seven months below those reported by Nolan and Ashcroft.

The results of research conducted over a 40-year period suggest that blind children are comparable to the sighted in their ability to reason mathematically, but that their ability to calculate is significantly below that of the sighted. However, there is the suggestion that it may be possible to improve the calculation achievement of blind children. Hayes (1944) reported that two schools scored at the seeing norm the first time calculation tests were administered. Another school, by placing greater emphasis

on mathematical instruction, brought mathematics achievement almost to the sighted norm. Nolan (1959) found a significant difference in mathematics achievement between schools. Brothers (1972) found the computational technique used by students to be a factor in their calculation achievement. Brothers' findings are of particular interest since the intent of this study is to provide instruction in the use of a calculating device.

The Abacus

The abacus, whose origin may be traced to the development of the number system, developed as a complement to early man's understanding of the number system and was widely used at a time when his ability to use and record written symbols was very limited. Paradoxically, this ancient device has within recent years been used by the blind who, like early man, are limited in their ability to manipulate and record written symbols.

Historical Development

The origin and development of the abacus is closely related to the development of the number system. Perhaps a shepherd moved stones from one pile to another as his sheep left the fold each day. The shepherd who understood the concept of group number could represent a fixed number of sheep, perhaps 10, by moving a given stone from one place

to another. The use of counting boards, flat boards with markings to indicate place, and the abacus, on which beads are moved along rods, is based on the concept of group number.

The Latin word "abacus" comes from the Greek "abax" which referred to any flat surface. "Abax" has been related to the Semitic "abaq" meaning dust. Abacus could mean dining table, sideboard, gaming-table, or a flat decorative wall panel. It also meant a reckoning board or table. Several authorities (Cranmer, 1963; Him, 1964; Kojima, 1954) suggested that abacus referred to a table strewn with sand, but Pullan (1969) noted that the abacus is ancient enough to have been invented in some Eastern country when it was customary to sit on the ground rather than at a table. If Pullan is correct, dust could have been associated with abacus calculation without being necessary to it.

The term abacus may be used with reference to a variety of calculating devices. Smith (1969) reported that the first abacus was probably a board on which a Babylonian spread sand so that he could trace letters. The Hindu culture calculated by etching figures into a clay tablet strewn with purple sand (Abacus, 1967). A form of abacus was used on the American continent. The Incas tallied by tying knots in a cord and the Mayas by threading grains of corn on a strand in groups of 10. Three types of abaci were used in

ancient Rome: the sand board or wax tablet, the marked table on which counters were moved, and a grooved frame on which counters were free to slide. Pullan (1969) discussed in detail the European use of jetons (counters) on tables marked with lines to show place value. Because merchants calculated on lined tables, we use the word "counter" to mean a flat surface over which business is transacted.

One of the earliest references to the abacus in the Western world is that of Herodotus (485-425 B.C.) who mentions calculation with pebbles and gives examples of problems worked (Pullan, 1969). What is apparently a Greek computing table was found in the nineteenth century on the island of Salamis (Menninger, 1969). Calculation with counters passed from Greece into Rome and was practiced in Europe until the advent of Arabic numerals in the seventeenth century.

The usual method of calculation during the Roman period was with loose counters, but four bead-frame abaci have been found. One has been lost but the others are in the British Museum, the Bibliotheque Nationale in Paris, and the Museo Nazionale Romano in Rome. They are all small enough to be held in one hand while beads are moved with the other.

There is a reliable account of the origin of the Chinese abacus in Mathematical Treatises by the Ancients written about 200 A.D. (Kojima, 1963). The similarity of the Roman

and Chinese abacus and the method of calculation suggests that the abacus was brought into the Orient through trade with the Romans. The Chinese abacus, called suan-pan, has two sets of beads on each column separated by a bar which extends the full length of the abacus. Arthur (1910) compared the two beads above the bar to a man's hands and the five below to his fingers. Each bead below the bar stands for one and each bead above the bar for five.

Kojima (1963) reported that the abacus was probably known in Japan as early as the seventh century. The oldest documentary evidence dates from the sixteenth century, but the Japanese abacus or soroban did not come into common use until the seventeenth century. Since then, three types of abaci have been used in Japan. One Japanese variation, like the Chinese suan-pan, had two five-beads and five one-beads. Until 1868 this abacus was used concurrently with a soroban which had one five-bead and five one-beads. Since about 1940, the Japanese have used an abacus with one five-bead and four one-beads. The bead arrangement on an abacus used by the blind is the same as that on the modern form of the Japanese abacus.

Application to the Education of the Blind

Japan. When education of the blind began in Japan in 1878, instruction in soroban calculation was an important

part of the general school curriculum; hence, soroban instruction became part of the curriculum for the education of blind children. Abacus instruction begins in the second grade in Japanese schools for the blind. A variety of abaci modified to meet the needs of the tactual operator have been used by the blind in Japan. In 1951 Koichiro Takeda, author of Teaching the Effective Use of the Soroban to the Blind Especially Addition and Subtraction, designed an abacus for blind students.

During the early part of this century Japanese interest in abacus calculation was stimulated through special courses and contests. In 1944 the Japanese Chamber of Commerce and Industry adopted a plan to conduct a national abacus calculation proficiency test (Yamazaki, 1966). Takeda (no date) reported that the first all-Japan soroban contest among blind children was held in 1950. In 1965 the Japanese Chamber of Commerce and Industry with the National Society of Principals of Schools for the Blind held the first Abacus Proficiency Test for the Blind (Japan National Committee, no date). The proficiency test included problems in addition, subtraction, multiplication, division, and mental calculation. It was offered at five levels, A through E. The proficiency of those qualifying in Class A is equal to or higher than that of third-class sighted operators. According to the Central Committee of the Federation of Abacus Workers average students

who begin studying the abacus in their teens should be able to pass the examination for a third-class license by practicing 1 hour a day for 6 months (Kojima, 1954). From 1965 to 1969, 4,040 people participated in the proficiency test for the blind (Japan National Committee, no date). Of that number more than 1,400 qualified at some level. The 18 blind people who qualified as Class A operators could be expected to add a column of ten, five- and six-digit numbers in 30 seconds if the problems were read orally (The 1st abacus proficiency test, 1966).

Western Europe. The use of the abacus in schools for the blind in Western Europe began in Denmark in 1959 and has since spread to Sweden, Norway, Germany, Great Britain, and the Netherlands. Paske (1967) described Danish, Swedish, and Norwegian abaci as well as the Takeda abacus and the Cranmer abacus. Paske designed the Danish abacus. The abacus was demonstrated at the International Council of Educators of Blind Youth in Hanover, Germany, in 1962.

United States. Rickard (1948) reported that Newel Perry, a teacher of mathematics at the California School for the Blind in 1912, expressed an interest in the abacus as an arithmetic aid for the blind. Perry adapted the device and used it as a recording device when performing mental arithmetic.

Rickard with the assistance of Howard C. Terry (1947,

1948) designed several working models of abaci which could be used by the blind. Creating enough friction to hold the beads in place was a primary problem. Rickard (1947) suggested that this might be done by creating external friction between the beads and some other substance, by modifying the rods, or by changing the structure of the beads. Rickard and Terry experimented with changing the size of the rods and modifying the structure of the beads. They did not attempt to use friction to create tension--the principle on which the Cranmer abacus is based.

The final product of Rickard's study was a lever abacus which represented a change in the structure of the beads. The levers were beveled to facilitate rapid detection and manipulation. They were moved by pressing the pointed end with one's fingers. The levers which were arranged like the beads on a Japanese abacus. Each abacus had nine columns and there was a sliding decimal point attached to each frame. Because students reported that less friction was necessary as they became accustomed to abacus calculation, the lever abacus was designed so that the friction level could be adjusted to meet the needs of each operator. Rickard (1948) believed that the abacus would: serve the needs of the blind as effectively as pencil and paper, promote accurate and more rapid calculation, and relieve the mind of memory work associated with computation. He reported plans (1947, 1948)

to construct enough lever abaci to permit experimental use in the classroom, but there is no record of such a study (Wilcox, personal communication, June 15, 1971).

Haas (1963) described abaci to be used for tactile analysis of the number system and for calculation. On all the abaci designed by Haas, numbers were recorded by moving counters up and over wire loops. On one abacus (1963, 1965) the beads were arranged in the Japanese pattern.

T. V. Cranmer's interest in the abacus began when he saw Japanese businessmen in Los Angeles using it (Cranmer, 1963). Cranmer discussed the potential of the abacus with the American Printing House for the Blind which led to a mutual effort to develop an abacus that could be used by the blind. The Cranmer abacus is a pocket-sized device adapted from the standard Japanese soroban. There are five beads on each of its 13 rods. A bar which extends the full length of the abacus separates the beads so that on each rod there are four beads below the bar and one bead above the bar. The back of the abacus is lined with foam padding which creates friction and limits the movement of the beads so that they can be examined by the tactual operator. It is possible to perform all of the basic arithmetic operations on this device (Bruce, 1963).

The abacus has been available from the American Printing House for the Blind since September, 1963. Production

figures from the 98th, 99th, 100th, and 101st Annual Reports to the Board of Trustees of the American Printing House for the Blind indicate that from 1963 through fiscal 1971, 28,558 abaci were sold. More than 8,500 abaci were sold during fiscal 1971. The abacus is one of the most popular items sold by the American Printing House for the Blind.

In the summer of 1964 the Lions Club of Lexington, Kentucky, and the Library Society for the Blind of Cincinnati, Ohio, sponsored what Gissoni (1965) believed was the first abacus course ever taught in the Western world. Thirty teachers of the blind from 17 states participated in the course. Fred Gissoni of the United States, K. Takeda and T. Horie of Japan, and Valdemar Paske of Denmark, all of whom are closely associated with abacus calculation by the blind, were present. Since 1964, workshops for teachers of the blind have been held at several teacher training institutions and schools for the blind.

The abacus is widely used in educational settings which serve blind students. Lewis (1970) conducted a survey of residential and public school programs to determine what aids were used in computation. There were 31 responses from residential schools and 43 responses from public school teachers. She found that the Cranmer abacus was used in 87% of the residential schools and 35% of the day classes reporting. One state was responsible for 47% of the abacus use in

day school programs. Fifty percent of the residential schools and 73% of the day school programs reporting had introduced the abacus by the end of the third grade. Hattendorf (1971) estimated that 20 residential schools used the abacus beginning in the fourth or fifth grade. The Hadley School for the Blind has offered an abacus course since 1964. Several residential centers include abacus instruction in their training of the adult blind. In less than 10 years the abacus has come to play an important role in the mathematical training of blind students.

Effect on the Calculation Performance of Blind Children

Educators consider the abacus to be of value to children, but there is relatively little empirical evidence to support that judgment. There are several descriptive accounts which suggest that the abacus has a positive effect on the calculation performance of blind students (Gissoni, 1963b, 1965; Hattendorf, 1971; Kohr, 1963; Lewis, 1968; Lewis & Coker, 1971). Three studies (Brothers, 1972; Davidow, Germain, & Rice, 1964; Nolan & Morris, 1964) which considered the relationship between skill in abacus calculation and achievement in computation have been reported.

The respondents in Lewis's 1970 survey ranked computational aids according to their value to students. The Cranmer abacus was rated first by 40% of the educators,

the braillewriter by 35%, mental calculation by 20%, and the Taylor slate by 5%. There was variation in the second, third, and fourth value rankings, but the first value rankings were identical for local day school and residential school respondents.

Kohr (1963) reported his observations regarding the reaction of junior high students to the abacus. He noted that at times the children seemed to learn faster from other students than from him and that some of them developed a method of their own for understanding the number relationships involved. The children were surprised and proud of their unexpected ability to add huge, complicated numbers or long columns of figures. The students did more problems in class and worked problems they would not have attempted earlier. The quality of homework papers improved and the classroom atmosphere was less tense. Kohr concluded that the abacus would eliminate students' fear of arithmetic.

Lewis and Coker (1971) reported that students at the Tennessee School for the Blind showed a high degree of interest when the abacus was introduced during the 1967-68 school year. The students worked in small groups for fun. Contests were an outgrowth of student interest. There was competition which involved the entire student body. In one free-style competition the contestants used either the Taylor slate, braillewriter, Cranmer abacus, or a device

of their choice. There were two people in each category. Lewis and Coker reported, "There were striking differences in speed with the abacus being much faster than any other method (p. 42)." There was also a contest between a team from the Tennessee School for the Blind and a team at the same grade level in a public school. Lewis (1968) reported that competition with a public school was gratifying and that it would never have been attempted with another means of calculation.

In April, 1970, six teachers concerned with the mathematics education of blind children and two representatives from the staff of the American Printing House for the Blind participated in an Institute on Instructional Materials Development in Mathematics for the Visually Handicapped (Franks, 1970). The report from that institute noted that the abacus may be used for computing with fractions, solving proportions, extracting square roots, and computing with logarithms as well as for computational tasks ordinarily taught in the early grades. It was recommended that the calculaid, cubarithm, Texas slate, and Taylor slate be replaced by the abacus and that the abacus be accepted as the basic tool for mathematical computation.

Davidow, Germain, and Rice (1964) reported the results of a research project with fifth and sixth graders at Perkins School for the Blind who were grouped according to

intelligence. The project was designed to consider whether abacus calculation was more accurate or faster than mental arithmetic. The mental arithmetic group also used braillewriters. Both groups worked one-half hour per day for 4 months with rotation of teachers and class time. Oral pretests and oral posttests were used to compare the two groups. "The results seem to indicate a definite improvement in accuracy and probably improvement in speed for those using the abacus (p. 103)."

Nolan and Morris (1964) conducted a study in which 42 students in grades 7B through 9B at Perkins School for the Blind were taught abacus calculation over a period of 8 months. The students ranged in age from 12 years to 16 years and 7 months and in IQ from 82 to 135. The children were pretested using the Advanced Arithmetic Computation Test, Form M of the SAT which was modified so that it included only problems in addition, subtraction, multiplication, and division of whole numbers and decimals. Form Am of the Madden-Peak Arithmetic Computation Test was also used in pretesting. The children were tested in grade groups by the usual braille procedures and computed with a method of their choice. The maximum time allowed was 40 minutes.

Posttests administered at the end of 4 months with different forms of the SAT and the Madden-Peak Tests showed a 27% improvement in the ability of students to handle the more

difficult material on the Madden-Peak but no improvement on the SAT. The forms of the SAT and of the Madden-Peak used in pretesting were administered as posttests after 8 months. This evaluation showed 33% more problems correct on the SAT and 66% more problems correct on the Madden-Peak. The authors noted that the Madden-Peak contains more difficult material than the SAT and suggested that the improvement was due to more than the normal acquisition of arithmetic skills. The low correlation between intelligence and improvement from test one to test three was interpreted to suggest that the abacus may be effective in improving computation skills over a wide range of intelligence. Nolan and Morris suggested that a more sophisticated method of computation allowed "the blind children in the study to operate successfully at a much higher level of difficulty, without necessarily increasing basic knowledge of arithmetic (p. 17)."

In his study of the computational achievement of braille students Brothers (1972) explored the relationship between achievement and computational technique used by students. He found that "once introduced the rate of achievement for students using the abacus was greater than any other group (p. 6)." Brothers suggested that third- and fourth-grade students should use either the Numberaid or some other concrete manipulative device, that there be less emphasis on the use of the braillewriter as a

computational device, and that by the end of sixth grade all students be given the opportunity to become proficient in the use of the abacus.

The results of a survey of educators of blind children, the recommendation of experts, and systematic research support the conclusion that skill in abacus calculation has a positive effect on the calculation performance of blind children. However, Nolan and Morris (1964) suggested that prospective users not expect an immediate and radical improvement in computation ability. They also noted that success required a specific technique, knowledgeable instruction, and much practice. Adequate instructional materials are a necessity if the abacus is to have a maximum effect on the computation achievement of blind children.

Teaching Abacus Calculation

Instructional Materials

To be effective instructional materials must present the necessary information in an orderly sequence and must use language appropriate to the educational level of the student. The rate of presentation must be commensurate with the student's ability to absorb information and there must be ample opportunity for the student to apply his knowledge. There should be objective evaluation of the student's progress. Published and unpublished materials about the abacus and abacus calculation were reviewed in an effort to identify

the characteristics of each that would make them of value to teachers in introducing blind children to abacus calculation.

Published materials. Two scholarly volumes, Number Words and Number Symbols A Cultural History of Numbers (Menninger, 1969) and The History of the Abacus (Pullan, 1969), provide a great deal of interesting background information but would be of little value in teaching calculation. The Abacus: A Pocket Computer (Dilson, 1968) presents the background and development of the abacus in language appropriate for students between the fifth and eighth grade and would be a good choice for school libraries. Having described abacus arithmetic as more fun than pencil and paper arithmetic, Dilson would probably agree that he was more interested in presenting the abacus as a fascinating tool than in teaching calculation.

The discussions by Crook (1958) and Him (1964) which describe calculation on the Chinese abacus would be less valuable to instructors who teach calculation on the Cranmer abacus than those of Japanese authors. In The Japanese Abacus Its Use and Theory, Kojima (1954) presents a brief history of the abacus and a comprehensive discussion of competition between the abacus and the electric calculator. Procedures for adding, subtracting, multiplying, and dividing on the abacus are described with an emphasis on bead manipulation technique. There is also a discussion of

mental calculation based on visualization of the associated bead manipulation. The problems in this volume were written and arranged so that the student might compare his skill with the skill of those who apply for a Japanese abacus license. They contain 15 numerals of from two to five digits. This volume would be helpful to the teacher in developing her own knowledge of the abacus, but the problems it contains are too complex for children beginning to use the abacus. In 1963 Kojima wrote Advanced Abacus Japanese Theory and Practice as a sequel to his earlier book. This book assumes knowledge of the fundamentals of abacus calculation and would not be appropriate as an introduction to abacus calculation.

The Japanese Abacus Explained (Yoshino, 1963) covers the four fundamental operations and summarizes the procedures for addition and subtraction in a Table of Addition and a Table of Subtraction which are similar to Davidow's (1967) presentation of complementary numbers. Yoshino's description of bead manipulation is identical to that of Kojima (1954). A teacher might use this volume which was written for high school and college students.

The American Printing House for the Blind publishes Using the Cranmer Abacus for the Blind (Gissoni, 1964) and The Abacus Made Easy (Davidow, 1967). Both books describe abacus calculation with reference to the Cranmer Abacus for

the Blind. Gissoni, who was associated with T. V. Cranmer in the initial development of the Cranmer abacus, deals with the four fundamental operations and, in a limited way, with decimals, fractions, and the extraction of roots. Money and change-making are used to explain the logic of addition and subtraction on the abacus. There is an explanation of the use of the abacus as a calendar. The limited number of practice items and the language level of *Using the Cranmer Abacus for the Blind* are such that it would not be of significant value to teachers in presenting abacus calculation to children.

Davidow taught blind children abacus calculation with *Using the Cranmer Abacus for the Blind* but felt the need for a simpler text and developed her own. She (1967) deals with addition, subtraction, multiplication, division, fractions, decimals, percentage, and the extraction of square roots. Davidow's presentation is more direct and readable than that of Gissoni but would be of limited value as an instructional aid because it contains only a small number of problems in which one might apply the principles taught.

Students who register for the abacus course offered by the Hadley School for the Blind use a course outline written to accompany *Using the Cranmer Abacus for the Blind* (Gissoni, 1964). The *Course Outline* is currently being revised, but

will, like the current form, of the Course Outline consist of a series of multiple-choice questions (Hattendorf, personal communication, April 27, 1971). Each item begins with a specific problem and, in subsequent steps, the student is asked to choose answers which indicate the procedure used to reach a solution. The Course Outline was not written as an instructional aid for the classroom and would not be optimally effective as such.

Unpublished materials. Kopecky (1969) wrote Set 4; Clear 2 The Use of the Abacus at the Texas School for the Blind to explain addition, subtraction, multiplication, and division in simple, brief steps. The procedures for addition and subtraction are presented in a logical sequence based on the number of columns required to complete the calculation. Kopecky's manual, which he wrote for teachers at the Texas School for the Blind, does not contain any practice problems.

The author examined two unpublished papers (Fish, Forster, & Ford, 1966; Jacquat, 1969) which are alike in that the goal of each was to present abacus calculation to blind children in a logical and progressive sequence. Although it seems reasonable, the sequence in which Fish, et al. presented the rules of addition and subtraction was different from that used in any other materials which the author reviewed. The sequence of presentation used by Jacquat is comparable to that of Gissoni (1964), but her materials

show that she also studied Davidow (1967) and Yoshino (1963) in developing her presentation. Both papers contain a number of problems designed to accompany the verbal explanation. The problems are arranged so that to complete them one must use the principles taught at the preceeding level. Materials similar to those by Fish, et al. and Jacquat which combine the presentation and application of calculation skills in a logical sequence are of the greatest potential value to instructors who teach the fundamentals of abacus calculation.

Bead Manipulation

Abacus calculation requires four kinds of bead movement: lower beads toward the bar, lower beads away from the bar, upper beads toward the bar, and upper beads away from the bar. Both the Chinese and the Japanese (Crook, 1958; Dilson, 1968; Kojima, 1954; Yoshino, 1963) have concerned themselves with the most efficient use of the fingers in abacus calculation, and teachers of the blind (Davidow, 1967; Kopecky, 1969; Neuman, 1970; Paske, no date; Takeda, no date) have expressed opinions on the subject. An efficient and ordered procedure for bead manipulation may contribute to one's proficiency in abacus calculation and so ideas on the subject have been reviewed.

According to Crook (1958) the standard Chinese method of manipulating beads on the abacus is to move the lower beads with the thumb and index finger and the upper beads

with the middle finger. According to Dilson (1968) the Chinese use their index finger to move the upper beads and their thumb to move the lower beads.

The Abacus Made Easy (Davidow, 1967) contained recommendations for bead manipulation. Davidow referred to the index finger of the left hand as the helping finger and said that it should be placed immediately to the left of the right hand. Partially sighted children should keep their fingers in the lower portion of each column instead of placing them directly on the beads. The right index finger should be used to set beads to the bar. When adding (4 + 1) the right index finger should sweep across the bar from top to bottom. When adding (9 + 1) the left hand should set 10 before the right hand clears 9.

Kopecky (1969) recommended that the beads below the bar be set and cleared with the right thumb. The beads from above the bar should be set and cleared with the right index finger. Kopecky's presentation of addition and subtraction is ordered so that one adds and subtracts with numbers formed using only beads from below the bar before calculating with numbers involving beads from above the bar.

Paske in Introduction to the Use of the Japanese Abacus (no date) suggested that the standard operating position of the abacus be reversed. The part of the abacus with the most beads should be fartherest away from the blind operator

who moves the single bead with his thumb and the four beads with his index finger.

Takeda (no date) recommended that blind children use six fingers, the thumb and first and second fingers of both hands, when calculating on the abacus. One should set the numbers one through four with the thumb of the right hand. Ten, 20, and 30 should be set with the left thumb. Five should be set with the middle finger. Six, seven, eight, and nine should be set with the thumb and index or middle finger in a pressing motion. One through four should be cleared with the index finger and five with the nail of the index or middle finger. When clearing six, seven, eight, and nine, one should clear the five-bead before clearing a bead from below the bar. Takeda recognized that this procedure complicated manipulation and required a great deal of initial instruction, but felt the results (rapid mechanical manipulation) merited the effort. Takeda suggested activities for practice in bead manipulation.

According to Kojima (1954) experiments have shown the index finger and thumb of the right hand to be most efficient in moving beads on the abacus. He emphasized the importance of manipulation technique and described in detail the manipulation procedure to be used in a number of simple problems. One should move the upper bead with his index finger and the lower bead with either his thumb or index

finger. The thumb should be used to move beads toward the bar and the index finger to move beads away from the bar. Since it is the basis for the technique presented in the abacus program, simple applications of the Japanese technique are described below.

1. One should move a one-bead and a five-bead toward the bar simultaneously (2 + 6).
2. One should move a one-bead down before moving the five-bead up (8 - 6).
3. One should move a five-bead down before moving down a one-bead (4 + 1).
4. One should move up one or more one-beads before moving the five-bead up (5 - 1).
5. When adding, one should finish an operation in the ones column before moving a bead in the tens column (3 + 7).
6. When subtracting one should move a bead in the tens column before moving beads in the ones column (10 - 7).

Neuman (1970) maintained that abacus calculation should be carried out with "a minimum of manipulative movements consistent with the limits of a blind person's tactile perception (p. 162)." A sighted operator grasps the totality of bead placement in an instant, but the blind operator discerns only what is under his fingers at a given instant. Neuman

developed a procedure for bead movement which he believed allowed for maximum speed and a minimum shift of attention. For a totally blind abacist, the first step is to tactually determine the location of the beads; therefore, all bead movements should begin in the lower portion of the abacus. Since the five-bead may not be involved in calculation, time need not be spent in examining it.

The general rules which evolve from this principle are summarized below:

1. The thumb, having determined (with or without the assistance of the index finger) placement of the lower beads, should move the lower beads to the bar before moving the five-bead. Neuman said that this "might become a simultaneous neuromuscular motion for the experienced operator (p. 163)."
2. Lower beads are cleared before the five-bead is cleared.
3. Lower beads are set before the five-bead is cleared.
4. Lower beads are cleared before a five-bead is set.
5. All bead movements on a column should be completed before moving to the left.

There is a marked similarity in the principles of manipulation described by Kojima and those Neuman suggested for the blind operator. Both methods assume observation of bead placement prior to bead movement. The totally blind abacist

must perform both steps tactually and so Neuman made two changes in Kojima's approach. A one-bead and a five-bead are not necessarily moved simultaneously, and, when subtracting, one completes all movements in a given column before moving to the left. The revised abacus course which is currently being developed by the Hadley School for the Blind will teach the techniques of bead manipulation described by Neuman.

Kojima (1954) reported that experiments had shown the thumb and index finger of the right hand to be the most efficient in moving beads on the abacus. There are no known studies of bead manipulation by blind students. Educators of the blind (Davidow, 1967; Kopecky, 1969; Neuman, 1970; Paske, no date; Takeda, no date) have recommended various applications of the thumb and index finger of the right hand in bead manipulation. The technique described by Neuman seems to be of the greatest potential value to tactually oriented abacists. Before a final judgment about the effectiveness of any one method can be made, it should be established that a given technique can be taught.

Programmed Instruction

Programmed instruction "is what an effective teacher does intuitively (Taber, Glasser & Shaefer, 1965, p. 2)." The assumption is that if students are subjected to appropriate experiences in the proper sequence they will develop

a given proficiency. The selection and presentation of appropriate learning experiences, the development of a sequence of presentation, and a description of the desired proficiency are based on principles which have been described by behavioral psychology. These principles are applicable to any instructional situation but will be considered here as they apply to one instructional technique, programmed instruction.

1. Behavior is deliberate, observable action by a student. The goal of instruction should be some carefully specified, observable behavior.
2. A stimulus is that feature of one's environment which produces a given behavior. Prompts may be presented and later withdrawn as a student learns the meaning of a given stimulus.
3. A student's response is his reaction to a given stimulus. The goal of instruction should be to establish a predictable relationship between stimulus and response.
4. Reinforcement, the consequence of a student's response, is a critical factor in the modification and maintenance of behavior. Programmed instruction places strong emphasis on positive reinforcement--that is, a consequence which the student would consider desirable. Reinforcement should be

delivered immediately after the response is made.

Ashcroft (1963) maintained that the principles of programmed instruction were "compatible with the effective instruction of the blind (p. 207)." One characteristic of programmed instruction is that the information is presented in small units each of which is directed toward achievement of the program objectives. Samways, Andres, and Klaus (1964) suggested that an approach in which the information offered is limited to that which is necessary and relevant would be especially valuable to the blind because they are limited in their ability to scan either braille or recorded material. Ashcroft (1963) noted that special aids and devices might be more readily accepted by the blind if it were easier to learn their use and indicated that programmed instruction would be an appropriate medium of instruction. Ashcroft and Henderson (1963) wrote _Programmed Instruction in Braille_ as a means of teaching braille to sighted adults, and Rex (1970) used programmed instruction to teach braille contractions to elementary school children.

In an early study of programmed instruction for the blind Coffey (1970) concluded that: the blind do learn from programmed instruction; programs written for the sighted did not appear to produce optimal results; and programmed instruction should be written specifically for the blind. The nature of the content and mode of presentation and response are two

variables which may influence the effectiveness of programmed instruction with blind students.

The content of a program in abacus calculation seems ideally suited to the carefully ordered presentation of information in small steps which is characteristic of programmed instruction. Davidow (1967) said that, in learning to calculate on the abacus, "We must move slowly; the language must be simple; every step must be understood (p. VIII)." With programmed instruction one might control the level of difficulty of both the verbal instruction and the problems in which the student is expected to apply his knowledge. The consistent and frequent feedback which is characteristic of programmed instruction should assure that a student understands each successive step in the instructional sequence.

The manner in which programmed instruction is presented to blind children is a major consideration in the development of an instructional sequence. Samways, et al (1964) concluded with reference to both braille and taped presentation that "presenting programmed materials to the blind requires a restructuring of the contents of each program frame in terms of its length and the position of the response requirements (p. 12)." Coffey (1970) presented programs in English and algebra to junior and senior high school students in braille and on tape but did not find mode of presentation to be a factor in the effectiveness of these programs.

Mallinson (1967) found that braille was the most effective means of presenting a 50-frame science program to junior high school students.

Samways, et al. (1964) presented a 54-frame program in English Money to 20 high school and 20 adult braille readers in braille or on tape. They noted that for a taped presentation frames must be short enough to allow the student to recall all the necessary elements and that all information and cues necessary for a response must be presented before the response was called for. Data for the Ss' error rate and completion time were interpreted to suggest that taped presentation of programmed materials "can produce a relatively high level of criterion performance in less time than it takes to learn from the same programmed materials presented in braille (p. 11)."

Tobin, Clarke, Lane, and Pittam (1970) reported six exploratory studies of programmed learning for the blind. One is of particular interest here. In that study the authors presented a taped elementary science program to a group of 10 totally blind children. They concluded that "a significant amount of learning can be achieved using group methods of presentation that allow the teacher to retain control over the pace of learning (p. 20)."

Mode of response is also a significant variable in the development of programmed instruction for blind children.

Coffey (1970) found that high school students performed best when constructing braille responses but that junior high students were more successful when they responded orally. Coffey interpreted his findings to suggest that for younger students braille may not be a thoroughly learned skill. If Coffey's conclusions regarding the braille skills of junior high students are valid, they would seem to be even more applicable to intermediate grade students and, hence, of particular significance in the development of the abacus program. Samways, et al. (1964) felt that the more rapid performance of the Ss who used the aural edition of their program was a consequence of oral as opposed to braille response. The Ss in the study by Tobin, et al. (1970) responded by conducting simple science experiments and by constructing braille responses. The science program developed by Tobin, et al. is comparable to programmed abacus instruction in that both require a physical response to the stimulus.

Summary

This review of literature has concerned itself with four variables in the design of a program to teach blind children addition and subtraction on the abacus: mathematics achievement by the blind, the development of the abacus and its application to the education of blind children, the characteristics of instructional materials that might be used in introducing blind children to the abacus, and the

appropriateness of programmed instruction as a medium of instruction for the blind. The review suggested that there was some disparity between the ability of blind children to reason mathematically and their ability to calculate. The abacus is an ancient calculating device that has been modified to meet the needs of the blind operator. The results of limited empirical study suggest that it is an effective aid. Authorities have emphasized the need for consistent practice, but the instructional materials available are not the kind that would give the blind child enough orderly practice. Programmed instruction is characterized by systematic presentation and review with frequent feedback. Blind children do learn from programmed instruction. Therefore, it is reasonable to assume that they would profit from programmed instruction designed to teach addition and subtraction on the abacus.

APPENDIX B

QUESTIONNAIRE SENT TO EXPERTS WHO REVIEWED THE INTRODUCTION TO PROGRAMMED ABACUS INSTRUCTION

QUESTIONNAIRE SENT TO EXPERTS WHO REVIEWED THE INTRODUCTION TO PROGRAMMED ABACUS INSTRUCTION

1. Could a teacher individualize each child's physical orientation to the abacus more efficiently than was done in the program?

2. What is your reaction to the technique for bead manipulation presented in the program?

3. Does the program allow for an optimal amount of understanding versus rote response?

4. Is the content presented in a manner that would allow for the further development of abacus skills?

5. I have used the word "secret" to mean a rule of abacus calculation and the words "heaven" and "earth" to refer to beads above and below the bar. What is your reaction to these terms?

6. I have considered introducing the procedure for adding 39 + 11 in either of two ways. It might be introduced after both secrets (rules) for adding one had been presented or at the end of the addition portion of the program after all the rules of addition had been presented. Do you have a suggestion on this point?

7. I plan to present the rules of abacus calculation as complements of either 5 or 10, but would like to develop a special relationship between the groups of rules I have listed below.

Clear 4, set 5	Clear 4, clear 5, set 1 left
Clear 3, set 5	Clear 3, clear 5, set 1 left
Clear 2, set 5	Clear 2, clear 5, set 1 left
Clear 1, set 5	Clear 1, clear 5, set 1 left

This might be done by pointing out that one must always add 5 either by setting it directly or by clearing 5 and setting 1 left. This may require an additional step in the logic process, but it seems to have some advantage from a manipulative point of view. What do you think?

8. I plan to present the program on tape and have the students write their answers. Do you feel that this mode of presentation and response would be optimally effective for blind children? I would appreciate any comment you might have about the appropriate time interval between presentation, student response, and reinforcement.

Any additional comments or criticisms will be appreciated.

APPENDIX C

DESCRIPTIVE DATA FOR INDIVIDUAL LESSONS

DESCRIPTIVE DATA FOR INDIVIDUAL LESSONS

Table 12

The Lessons

Lesson	No. of Frames	No. of Sections	No. of Criterion Items	No. of Unacceptable Frames	No. of Unacceptable Sections	Lesson Acceptable
1	18	2	6	1	1	No
2	24	1	10	0	0	Yes
3	24	2	12	0	0	Yes
4	21	1	16	1	0	No
5	16	1	16	0	0	Yes
6	14	3	15	0	0	No
7	16	1	16	0	0	Yes
8	12	3	15	0	0	Yes
9	19	5	31	0	0	Yes
10	20	2	15	0	0	Yes
11	23	4	15	2	1	No
12	20	4	15	0	0	Yes
13	21	4	15	0	0	No
14	24	3	15	0	0	Yes
15	21	3	31	1	0	No
16	19	3	15	0	0	Yes
17	15	3	15	1	1	No
18	15	3	15	0	0	Yes

Table 12 (continued)

Lesson	No. of Frames	No. of Sections	No. of Criterion Items	No. of Unacceptable Frames	No. of Unacceptable Sections	Lesson Acceptable
19	19	3	15	2	0	No
20	21	3	15	4	0	No
21	23	3	15	2	0	No
22	20	3	15	2	2	No
23	22	3	15	1	0	No
24	24	3	15	3	2	No
25	20	3	15	1	0	No
26	20	2	15	3	1	No
27	18	2	15	2	1	No
28	15	2	15	1	0	No
29	15	2	15	0	0	Yes
30	22	3	15	3	1	No
31	19	3	15	0	0	Yes
32	19	2	15	2	1	No
33	17	2	15	2	0	No
34	17	3	15	1	1	No
35	16	3	15	0	1	No
36	17	3	15	0	0	Yes
37	13	2	15	0	0	Yes
38	18	2	15	7	1	No

Table 12 (continued)

Lesson	No. of Frames	No. of Sections	No. of Criterion Items	No. of Unacceptable Frames	No. of Unacceptable Sections	Lesson Acceptable
39	14	2	15	1	0	No
40	14	2	15	2	0	No
41	17	2	15	0	0	Yes
42	19	2	15	3	0	No
43	15	2	15	2	0	No
44	17	2	15	4	1	No
45	14	2	15	5	1	No
Total	827	114	693	59	16	16 Acceptable

Table 13

Median Completion Time in Minutes Per Lesson

for Subjects Who Completed the Program

Lesson	Median	Range
1	12.5	9-17
2	22	14-25
3[a]	17.5	13-20
4	20	11-26
5	16	7-20
6	21	14-32
7	15	10-22
8	19	14-25
9	42	35-62
10	22	15-32
11	31	24-44
12	26	21-40
13	24.5	21-36
14	33	23-35
15	38	29-51
16	24.5	20-36
17	28	21-35
18	24	18-27
19	25	18-30
20	25	24-39

Table 13 (continued)

Lesson	Median	Range
21	26.5	20-32
22	29.5	23-41
23	25.5	20-29
24	32.5	28-34
25	27	21-35
26	29.5	24-40
27	25	21-27
28	16.5	14-20
29	22.5	15-24
30	26	21-35
31[a]	24	19-28
32	26	21-28
33[a]	24	21-29
34	24.5	21-29
35	23	19-30
36	25	20-28
37[a]	19	18-30
38	23	22-25
39[a]	20	18-24
40	23	18-30
41	23.5	19-35

Table 13 (continued)

Lesson	Median	Range
42	28	22-45
43	24.5	23-32
44	26	20-30
45	29.5	25-37

[a] Median time for Lessons 3, 31, 33, 36, 37, and 39 is based on seven scores.

APPENDIX D

SUMMARY OF REVISIONS FOLLOWING FIELD TEST

SUMMARY OF REVISIONS FOLLOWING FIELD TEST

No change was made in the 16 lessons in which every frame and section met the standard of acceptability. Those lessons have been marked with an asterisk (*) as they appear in Appendix E. The changes made in the remaining 29 lessons involved unacceptable frames and unacceptable sections with particular reference to the unacceptable frames in those sections.

Frame Revisions

Each of the unacceptable frames was revised and classified according to the action taken. Data summarizing the manner in which each of the unacceptable frames was revised are reported in Table 14. The largest category was those frames omitted from the revised form. A review of data on the number of numerals and the number of rules in the acceptable calculation sequences as compared with the unacceptable calculation sequences suggested that the number of numerals and the number of rules applicable were factors in the difficulty of the unacceptable calculation sequences. The unacceptable calculation sequences were shortened. A number of unacceptable frames were modified as a consequence of revisions within the section of which they were a part. The revisions were associated with some modification in a frame or frames closely related to the unacceptable frame.

Table 14

Summary of Frame Revisions

Action	Number
Omissions	16
Calculation Sequence Shortened	10
Section Revision	9
Student Language	7
Frame Clarity	6
Calculation Simplified	5
Errors Corrected	4
Sequence Changed	3
Special Case	1
Total	61[a]

[a] The total number of frame revisions reported above is greater than the number of unacceptable frames because two acceptable frames were omitted in conjunction with the use of student language. (See page 115.)

The Ss were able to state the rules of abacus calculation but frequently did so in their own way rather than in the language of the program. For example, six of the eight Ss reported the rule for adding one as "Clear 9, set 1 left." rather than "Clear 4, clear 5, set 1 left." as it was presented in the program. The wording used by the Ss was accepted as a reflection of their understanding of the

relationship between numbers on which the rules of abacus calculation are based. The language which the Ss employed in stating the rules of abacus calculation was used in the revised program. Two acceptable frames were omitted as a result of the change in wording. In several instances a phrase was added, deleted, or reordered to clarify the content of a frame. A few frames were found to contain an error or were reordered within the sequence of instruction. The number of numerals, the number of digits, and the number of rules applicable in each of the unacceptable calculation frames were noted. Those frames were shortened when that review indicated that the unacceptable calculation frames were more complex than the criterion items in the section of which they were a part.

Section Revisions

The largest number of sections requiring revision were those in which criterion performance was acceptable but frame performance unacceptable. With one exception, all of the sections in which frame performance was unacceptable contained one or more frames that were unacceptable. Hence, there is some overlap between the description of frame revisions and the description of section revisions. Five of the unacceptable frame sequences were relatively short so that revision was most often associated with a single frame.

The ultimate test of abacus skill is one's ability to calculate. Therefore, in those cases where criterion performance was acceptable but frame performance unacceptable, it was assumed that the instructional sequence was more complex than necessary. The instructional sequence in those lesson sections was simplified in one of several ways. Three frames were omitted and five calculation frames shortened. For four frames, the sequence of presentation was changed; two frames were revised without changing the total number of frames in the section of which they were a part. Four frames were added and errors were corrected in three frames. The language used by the Ss was used in revising two frames, and the explanation offered in one frame clarified.

One section in which criterion performance was acceptable, but frame performance unacceptable, was given special consideration. The standard of acceptability was that at least six of the eight Ss who completed the program would score 75% or higher on the frames in a section, but since three Ss each missed one frame in that section, it fell below standard. Since every Ss whose frame performance was unacceptable scored above standard on the criterion problems, no change was made in any part of that section.

For two sections frame performance was acceptable, but performance on the criterion items was not acceptable. The

discrepancy in performance was interpreted to suggest that the frames did not teach the necessary skills. In one section the sequence of presentation was changed. A review of performance on the second section showed that four Ss missed the same criterion problem. One frame was added with special reference to that criterion problem.

In two of the lesson sections, neither the frames nor criterion problems were acceptable. In one of those sections, an unacceptable frame and three criterion problems were revised so that they conformed more closely to the skills which should have been taught in that section. In the other, the criterion problems were retained but the instructional sequence revised to deal specifically with the skills necessary to complete the criterion problems. Three frames were added and another revised. The three unacceptable frames in that section were retained with the hope that the additional explanation would raise them to the standard of acceptability.

Revisions Not Covered by Data Analysis

The comments and reaction of the Ss who completed the program seemed to warrant several changes in the manner that Ss responded to the program. For some frames, the Ss were asked to mark one of three possible answers, but during the field test, they either asked what kind of mark they should

use or marked in their own way. To eliminate confusion and establish a consistant pattern, instructions in the revised form specify that the answers be circled. In a number of frames, the *S*s were asked to indicate what number could be added with a given rule of abacus calculation by displaying that number of fingers. One *S* suggested that it would be easier to respond by setting that number of beads on the abacus. Several *S*s seemed to have difficulty extending their fingers; therefore, in the revised form, the instructions specify that one respond by setting the appropriate number on the abacus. Originally the *S*s were asked to raise a hand when they heard a certain statement. Instructions in the revised form specify that one not respond until all the options have been read.

Each lesson in the abacus program was numbered so that it could be readily identified. The first frame in six of the lessons explained to the *S* that he was beginning Lesson ____. Since the references to lesson number caused undue concern among the *S*s regarding their place in the instructional sequence, they were eliminated in the revised form.

APPENDIX E

PROGRAMMED INSTRUCTION IN ADDITION AND SUBTRACTION ON THE ABACUS--REVISION FOLLOWING FIELD TEST

PROGRAMMED INSTRUCTION IN ADDITION AND SUBTRACTION ON THE ABACUS--REVISION FOLLOWING FIELD TEST

Instructions to Reader

Presentation of this program requires the presence of a live reader to monitor and reinforce student responses. To the right of each frame is the word, phrase, or number that is the correct response for that frame. The letters "OBV" indicate that the reader should observe whether or not the student performs the designated task correctly. The student is asked to respond to some frames, for example, Frame 7 of Lesson Nineteen, by stating a rule of calculation. For those frames any phrase used by a student which expresses the correct idea may be considered acceptable. The problems presented in the calculation frames are written to the right of the frames. Statements which may follow each response have been included in the program as it is reported here.

The reader should adjust his reading rate to the pace at which each student is able to respond to each frame and criterion item in the program. This is especially important for those items which involve calculation. For example, Frame 17 of Lesson Twenty-five says, "Set 674 on your abacus. To that number add 765." In reading the number 765 the reader may pause to allow the student to add 700 before reading 65. Likewise, the reader may pause so that the student can add 60 before 5 is read.

The content of the Criterion Checks in Lessons Three, Four, Five, and Seven and in Section A of Lessons Nine and Fifteen is such that they should be administered individually. So that they may be administered to as many as three students simultaneously, those lessons contain three different sets of Criterion items.

Instructions to Students

I have just given you an abacus which is a tool you can use to work arithmetic problems. I have also written some lessons that should help you learn how to add and subtract on the abacus. What you should know in order to use an abacus has been divided into small steps and easy questions. There are several ways that you may answer a question. Some of the time you can answer by doing something with your abacus. Some of the time I will ask you to raise your hand or hold up your fingers to show that you know the answer. If the answer to a question is either "Yes" or "No," you should put your finger on one of the two cards that is taped to your desk. Find the cards on your desk (pause). When the answer is a number you have recorded on your abacus, move your fingers away as soon as you have finished so that I can see your answer. Some of the time I will ask one of you to tell me the answer to a question. In that case we will take turns. I will help you know whether you have answered the questions correctly. If you would like me to repeat a question or problem, quietly raise your hand. You should be sitting so that your desk is directly in front of you with your feet under it. Are there any questions?

LESSON ONE

1	The abacus has a smooth surface and a beaded surface. The back of the abacus is flat and smooth. Moving the beads up and down is a way of counting on the abacus. Put your abacus flat on your desk so that it will be easy to move the beads.	OBV
2	Does your abacus have three or four sides? You should show that you know the answer to that question by holding up either three or four fingers.	4
3	Now, here's another question for you. Are all four of the abacus's sides the same length?	NO
4	There is one word that should be used to finish this sentence. I would like one of you to tell me what you think that word is. The four sides of the abacus are not all the same length. Two of the sides are long and two are _____.	SHORT
5	Move your hand along the edge of your desk which is closest to you. It is very straight. The four sides of your abacus are also straight.	OBV
6	Place your abacus so that one of the long sides is even with the edge of your desk. All of a long side of your abacus should touch the edge of your desk. You should be able to touch the edge of your desk and a long side of your abacus at the same time.	OBV
7	Now move the abacus away from the edge of your desk, but make sure that a long side still runs in the same direction as the edge of your desk.	OBV

8 There is a bar on your abacus which has dots and lines on it. The bar runs in the same direction as a long side of your abacus. There are beads on both sides of that bar. Find the bar and move your fingers over it. OBV

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9 There are more beads on one side of the bar than on the other. Turn your abacus so that the part with the most beads is closest to you. OBV

Criterion Check

I'm going to read you a check list that should help you know whether your abacus is in the correct position. Listen carefully so that you can be sure your abacus is in the correct position.

Here's the check list.

1. Your abacus should be flat on your desk
2. A long side should run in the same direction as the edge of your desk.
3. Your abacus should be turned so that the part with the most beads is closest to you.

Place your abacus in the correct position and do not move it until I tell you to do so.

---

10 Now that you've learned how to place the abacus on your desk, you're ready to learn how to hold it. NONE

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11 If you are a braille reader, you probably read with your index fingers. You will also use them to do arithmetic on the abacus. Show me the index finger of your left hand. LEFT INDEX

No comment. or Show child his left index finger.

12	Some of the beads on an abacus stand for one number and some for another. It is important to be in the correct place on the abacus. When using an abacus you should use the index finger of your left hand to help you stay in the correct place.	NONE
13	The index finger of your left hand is called the tracking finger. When you are using an abacus, the tracking finger helps you stay in the correct place. I'm going to read two sentences. After I have finished, use one or two fingers to show me whether you think the first or second statement describes the finger you should use to keep your place on the abacus. 1. The tracking finger is the index finger of your right hand. *2. The tracking finger is the index finger of your left hand. No Comment. or The tracking finger is the index finger of your left hand.	ANSWER STARRED
14	Place your tracking finger on the upper right hand edge of your abacus.	OBV
15	Cover the abacus with your left hand as if you wanted to wrap your hand around it. Your palm should cover the beads.	OBV
16	The inside of your thumb should extend along the smooth bottom edge of the abacus close to the part where there are the most beads. Your thumb should point toward the right hand end of the abacus.	OBV
17	Wrap the other three fingers of your left hand over the top of the abacus so that there are beads under your palm.	OBV

Criterion Check

It's time for another check list to help you know you're holding the abacus correctly. Listen carefully.

Here's the check list:

1. Your left index finger should touch the top right hand edge of your abacus.
2. The thumb of your left hand should extend along the smooth bottom edge of the abacus. It should point to the right.
3. The other three fingers of your left hand should be wrapped over the top of the abacus so that there are beads under your palm.

Place your left hand in the correct position and do not move it until I tell you to do so.

(Each S̲ was scored for his performance on the above criteria and provided with individual guidance if he failed to meet them.)

LESSON TWO*

1	You know that the abacus is a tool which will help you work arithmetic problems. In this lesson you will learn to show the numbers 1, 2, 3, and 4 on the abacus.	NONE
2	You should begin this lesson with your abacus flat on your desk so that the part with the most beads is closest to you. Hold the abacus in your left hand with three fingers wrapped over the top and your tracking finger resting in the top right hand corner of the abacus. Your thumb should extend along the bottom edge. Hold your hand still so I can see that you understand. Good. or I will read the directions again.	OBV
3	The beads which one uses to count on the abacus are separated by a bar. We will call the two groups of beads the upper beads and the lower beads. The upper beads are above the bar. There are more beads in the part of the abacus which is closest to you. They are called the lower beads.	NONE
4	Use your tracking finger and the fingers of your right hand to help you find the beads closest to the right hand end of your abacus.	OBV
5	Place your left hand so that there is one column to the right of your tracking finger. Then hold still so I can see that you understand. Good. or I will read the directions again.	OBV

6	Find the right hand corner in the upper part of your abacus. How many beads are above the bar in the column next to the right hand end of your abacus? Use your fingers to show that you know the answer to that question. Good. or There is one bead in that part of the abacus.	1
7	How many beads are below the bar in the lower part of your abacus? Look only in the column next to the right hand edge. Use your fingers to show the answer to that question. Good. or There are four beads in that part of the abacus.	4
8	Look above and below the bar in the column next to the right hand edge of your abacus. Make sure that there are no beads touching the bar. I'm going to read you a sentence in which the last word is missing. Tell me what you think that word is. Your abacus says 0 because there are no beads touching the _______.	BAR
9	With the thumb of your right hand push one bead from below the bar up to the bar. Use your fingers to show me what number is recorded on your abacus. Good. or The number 1 should be recorded on your abacus.	1
10	Move the lower bead away from the bar. Now, I'm going to say three numbers. When I have finished, I'll ask one of you to tell me what number is recorded on your abacus. 1 0 4 Good. or Zero is recorded on your abacus.	0

11	When one bead is pushed up to the bar, your abacus has the number 1 on it. One at the time I would like each of you to set the number 1 on your abacus. You should use your right thumb.	OBV
12	You know that 1+1=2. Your abacus already has the number one on it. With your thumb push one more bead up to the bar. Use your fingers to show that you know what number is recorded on the abacus. Good. or The number 2 should be recorded on your abacus.	2
13	To record the number 2 on your abacus you pushed two beads up to the bar. When you push beads up to the bar you set them against the bar. You used your right thumb to set the number 2 on your abacus.	NONE
14	Check your abacus to be sure that two lower beads are pushed to the bar. Then, with your thumb set one more bead to the bar. I'm going to say three numbers. When I've finished I'll ask one of you to tell me what number is recorded on your abacus. 3 1 0 Good. or The number 3 is recorded on your abacus.	3
15	When no beads are set at the bar your abacus says 0. When one lower bead is at the bar it stands for the number 1. When two lower beads are set at the bar, they stand for the number 2. When three lower beads are set to the bar, your abacus says 3. Think about how you would show the number 4 on your abacus. Then I want to watch each of you one at the time record the number 4 on your abacus, you should use your right thumb.	OBV

16	Are there any more beads below the bar in the column next to the right hand edge that you could push up to the bar? No comment. or There are no more beads that you can push up to the bar.	NO
17	Move the four beads away from the bar so that your abacus will say 0. First your abacus said 4. Then you made it say 0. We say that you cleared 4 from your abacus.	OBV
18	Listen to these directions. Then I will ask each of you to follow them one at the time. With your right thumb set the number 2 on your abacus. Now use your right index finger to clear the two beads from your abacus. Your abacus should say 0.	OBV
19	Use your right thumb to push three beads up to the bar. Did you set 3 or clear 3? No comment. or You set 3.	SET 3
20	When you no longer need a number that is recorded on the abacus you clear it by moving beads away from the bar. Use your right index finger to clear 3.	OBV
21	Listen to these directions. Then I would like each of you to follow them one at the time. Set the number that is the sum of 2+2 on your abacus. Remember to set the beads with your right thumb.	OBV
22	Clear 4 from your abacus. Show me the finger you used to clear the beads. Good. or You should have cleared with your right index finger.	RIGHT INDEX

23 In talking about the abacus we have used five terms which you probably already knew. However, they have a special meaning when we use them to talk about the abacus. The terms were set, clear, lower beads, upper beads, and tracking finger.

I have given you a sheet with each of these words on it in five rows. You should circle the one word in each row that best answers the questions I am going to ask.

1. What is the name of the beads that are on top when you are using the abacus? UPPER BEADS

   You should have circled upper beads.

2. What word means to move a bead to the bar so that a number is recorded on the abacus? SET

   You should have circled set.

3. The bar on your abacus divides the beads into two groups. What are the beads below the bar called? LOWER BEADS

   You should have circled lower beads.

4. One finger of your left hand is used to help you keep your place on the abacus. What is its name? TRACKING FINGER

   You should have circled tracking finger.

5. You have learned to use your right index finger to move beads away from the bar. What word means to move beads away from the bar so that a number no longer appears on the abacus? CLEAR

   You should have circled clear.

---

24 At this point you know how to show the numbers 0, 1, 2, 3, and 4 on your abacus. I'm going to read you some very easy problems. After I have read each problem, I would like you, one at the time to record the correct answer on your abacus.

1. With your thumb set the number that is the sum of 2+1. 3

   Now use your index finger to clear 3 from your abacus.

2. Set the number that is between 1 and 3. Use your thumb to set the beads. 2

   Clear 2 from your abacus. Remember to use your right index finger.

3. Use your thumb to set the number that is between 0 and 2. 1

   Clear 1. Remember to use your index finger.

4. With your thumb set the number that comes after 3. Now clear 4 with your index finger. 4

   You should have set the four lower beads with your thumb and cleared them with your index finger.

5. Set the number that is the same as 3-3. 0

   Since that answer was 0 you will not need to clear your abacus. OBV

Criterion Check

The child's skill in the use of the abacus was evaluated at this point. He was checked to be sure that he was:

1. Holding his abacus properly.
2. Working only in the lower portion of the column adjacent to the right hand edge.

3. Setting with the thumb of his right hand.
4. Clearing with his right index finger.
5. Observing the one-to-one relationship between numbers and beads set to the bar.

Instruction to Students

I'm going to read five simple problems. I would like each of you, one at the time to record the correct answer on your abacus. I'll watch to see that you are using your thumb and index finger correctly?

GROUP A

1.	What is six divided by two?	3
2.	Set the number that tells how many ears you have.	2
3.	What is five minus four?	1
4.	Set the number that is between three and five.	4
5.	The football team did not make any points at all. Set the number that tells what their score was.	0

GROUP B

1.	What is nine minus 5?	4
2.	Set the number that tells you how many noses you have.	1
3.	Set the number that is between two and four.	3
4.	What is ten divided by five?	2
5.	Set the number that tells how many cavities you would like to have the next time you go to the dentist.	0

LESSON THREE*

1	So far you've learned to hold an abacus and to set and clear the numbers from 0 to 4. In this lesson you will learn to record the numbers 5, 6, 7, 8, and 9. By the end of this lesson you should be able to record all the numbers from 0 to 9.	NONE
2	Suppose that you had brothers who were twins. When I call your name, set the number that tells how many brothers you would have. (After everyone has set.) Use your right index finger to clear 2.	OBV
3	This time, when I call your name, set the number that tells how many children are in a set of triplets. (After everyone has set.) Clear 3 so that you will be ready to learn something new.	OBV
4	When I call your name, set the number 4 on your abacus.	OBV
5	Are there any more lower beads that you could move up to the bar? No comment. or There should not be any more beads you can set to the bar.	OBV
6	When I call your name, use your right finger to clear 4 from your abacus.	OBV
7	How many beads are in the upper part of your abacus? Look only in the right hand corner of your abacus. You should use your fingers to show that you know the answer to that question. Good. or There is only one bead in the upper part of your abacus.	OBV

8	Find the one upper bead that is in the right hand corner of your abacus. When I call your name, use your right index finger set that bead to the bar. Now your abacus says 5.	OBV
9	When I call your name, use your right index finger to clear 5 from your abacus.	OBV
10	You have just set and cleared the five-bead from your abacus. Show me the finger you used to set and clear that number. You should always set and clear the five-bead with your right index finger.	RIGHT INDEX
11	What number is recorded on your abacus when only the single upper bead is set at the bar? You should use your fingers to show that you know the answer to that question. Good. or The number 5 is recorded on your abacus.	5
12	You should always use your index finger when you clear the five-bead. Is the five-bear an upper bear or a lower bead? No comment. or The five-bead is an upper bead.	UPPER
13	It is important to remember how to move the beads on an abacus. This is a general rule that should help you remember how to use your fingers. In each column there are four lower beads and one upper bead. You should always use your thumb to set the lower beads. Any other time you move a bead on the abacus you should use your right index finger.	NONE

14	To move the lower beads on an abacus you might use either your thumb or your index finger. You should always clear lower beads with your index finger. Show me the finger you should use to set the lower beads? I agree. or You should set lower beads with your thumb.	THUMB
15	Set the five-bead with your right index finger. Now clear it with the same finger. You always use your index finger to clear a bead from the bar. Hold up the finger you would use to set the five-bead. I agree. or You should use your right index finger to clear the numbers from 1 to 4	RIGHT INDEX
16	The beads which you need to record 1, 2, 3, and 4 are in the lower part of the abacus. Hold up the finger which you should use to clear the numbers from 1 to 4. I agree. or You should use your right index finger to clear the numbers from 1 to 4.	RIGHT INDEX
17	You may use either your thumb or index finger to set a bead. Hold up the finger you should use to set the five-bead. I agree. or You should use your right index finger to set the five-bead.	RIGHT INDEX

I'm going to read you four statements that explain when to use your thumb. Only one of them is true. When I have finished, use your fingers to show me whether the first, second, or third statement is true.

1. You should use your thumb to set the five-bead.

18 2. You should use your thumb to clear the five-bead. ANSWER STARRED

*3. You should use your thumb to set 1, 2, 3, and 4.

4. You should use your thumb to clear 1, 2, 3, and 4.

You should use your thumb to set 1, 2, 3, and 4.

Criterion Check

I'm going to read some numbers which I would like you to set and clear from your abacus. I will watch to see that you are using your thumb and index finger correctly. I'll call on you when it's your turn to set and clear the numbers.

1. Set 1. Now clear 1.
2. Set 5. Clear 5.
3. Set the number 2. You may clear 2.
4. Set 3. Now clear 3.
5. This time set 4. Clear 4.
6. Show me 0. There is nothing to clear.

19	You know that 1+5=6. If you remember that, it will help you to know how to show 6 on your abacus. When I call your name, set 1 with your thumb. Then set the five-bead with your index finger.	OBV
20	You have just set the number 6 on your abacus. After I have read the instructions, I want each of you one at the time to clear 6 from your abacus. You should use your index finger to clear the lower bead and then the upper bead. Now clear 6 from your abacus.	OBV
21	Wait until I call your name to follow these instructions. With your thumb set 2 on your abacus. Now use your index finger to set 5. You know that 2+5=7. What number is recorded on your abacus? When I call your name clear 7. Use your index finger to clear the two lower beads and then the five-bead.	7
22	You learned to set and clear the numbers from 1 to 4 before you learned to set and clear the five-bead. Lower beads are used to record the numbers 1, 2, 3, and 4, and an upper bead to record the number 5. To record numbers greater than 5 it is necessary to use lower beads and upper beads. When both lower and upper beads are used to record numbers you should always set or clear the lower beads before you move the upper beads.	NONE

23	You know how to set 6 by thinking 1+5. You also know how to set 7 by thinking 2+5. Set 8 by thinking 3+5. When I call your name, you should use your thumb to set 3 and your index finger to set 5. Clear your abacus by clearing the lower beads and then the upper bead.	OBV
24	The lower beads on your abacus can be used to record the number 4. The upper bead stands for 5. When I call your name, set 9 on your abacus. Clear 9. Remember to clear the lower beads and then the upper beads.	OBV

Criterion Check

You've finished another abacus lesson. I'd like you to use your abacus to record the answers to some problems for me. I will ask five questions. As usual I'd like you to take turns recording the answer.

GROUP A

1. What number comes after five?	6
2. What is ten minus two?	8
3. What is fourteen divided by two?	7
4. What is the sum of three plus two?	5
5. What is three times three?	9

GROUP B

1. What is four plus four?	8
2. What is twenty-five divided by five?	5
3. What is fourteen minus seven?	7
4. What is six plus three?	9
5. What is twelve divided by two?	6

GROUP C

1.	What is nine minus one?	8
2.	What number comes after eight?	9
3.	What is three plus four?	7
4.	What is two times three?	6
5.	What is ten divided by two?	5

LESSON FOUR

1	So far, you've learned how to hold the abacus and to set and clear the numbers from 0 to 9. In this lesson you will learn to record the numbers from 10 to 49.	NONE
2	When I call your name, set 9 on your abacus in the column next to the right hand edge. Is there another bead in that column you could set to the bar?	NO
3	There are 12 columns on the abacus which you've not used. Should you look to the right or to the left of the column which says 9 to find a place for 10? Raise your right or left hand to answer that question. I agree. or The column for 10 is to the left of 9.	LEFT
4	Wait until I call your name to follow these instructions. With your thumb set one lower bead in the column immediately to the left of the column where you've already set 9. The bead which you just set says 10. Clear 9 from your abacus.	OBV
5	The column in which a bead is set to show 10 is called the tens column. When I call your name clear 10 from your abacus with the index finger of your right hand.	OBV
6	Set four beads from the tens column up to the bar, so that your abacus will say 40. There are no ones in forty so you will not need to use the column on the right hand edge of your abacus. Use the index finger of the right hand to clear 40.	OBV

7	Wait until I call your name to follow these instructions. With your thumb set two beads from the tens column to the bar. Then set 3 so that your abacus will say 23. The number 23 is a two-digit number. It has 2 tens and 3 ones, and so you used the tens column and the ones column to record it. Clear the 2 tens and 3 of 23.	OBV
8	Think about the number 34, but do not record it on your abacus. Does the three say that there are 3 tens or 3 ones in 34? No comment. or There are 3 tens in 34.	3 TENS
9	Suppose I asked you to write the number 34 on paper. Would you write the 3 tens or the 4 ones first? No comment. or You would write the 3 tens first.	3 TENS
10	Numbers are set on the abacus in the same order in which you would write them on paper. If you wanted to set the number 42 on your abacus, would you set the 4 tens of forty or the 4 ones first? No comment. or You would set the 4 tens first.	4 TENS
11	Think about how 42 would look on paper. Would the four be on the right or the left of the two? Raise your right or left hand to answer.	LEFT
12	The four which tells how many tens are in 42 would be written to the left of the 2. Is the tens column on your abacus to the right or left of the ones column? Raise your right or left hand to answer. No comment. The tens column is to the left of the ones column.	LEFT

13	Set 42 on your abacus. Did you remember to set the 4 tens before you set the two ones?	YES
14	Before you clear 42 check to be sure that the 4 tens of 40 are recorded in the column to the left of 2. Clear your abacus.	OBV
15	After I read the instructions I will call on each of you to set 33 on your abacus. Thirty-three is 3 tens and 3 ones. You should write the three which stands for 3 tens, or thirty, first, and so you would set thirty before you set the 3 ones. Clear 33 by clearing the 3 tens and then the 3 ones.	OBV
16	The number 24 has 2 tens and four. When I call on you, set 24 on your abacus. Remember to set the beads in the tens column before you set the four. Clear your abacus.	OBV
17	Use your thumb to set the 3 tens and 2 of 32. Did you set 3 in the tens column before you set 2 in the ones column? Clear your abacus.	YES
18	This time, when I call your name, set 37 on your abacus. First, set 3 tens. When you are ready to show 7 in the ones column you should think 2+5=7. Clear the three tens and seven of 37.	OBV

This time I would like each of you to set 39.

Now, when it's your turn clear 39. You
should clear with your index finger.
19 Remember to clear in the tens column OBV
before you clear in the ones column. Clear
the four in nine before you clear the five.

Set 46 on your abacus.
Did you set in the tens column before
20 setting in the ones column?

Clear your abacus. YES

21 At this point you should be able to record all the numbers from 0 to 49 on an abacus. I'm going to ask you to set and read some numbers for me without any directions. Before we do that, here's a check list that should help you review the best way to use an abacus.

1. Set the lower beads with your
right thumb. Any other time
you move a bead you should use
22 your right index finger. NONE
2. When you use both upper and lower beads to record a number, work with the lower beads first.
3. When a number has more than one digit, set the digits in the order in which you would write them.

Criterion Check

I'm going to read eight numbers to you. I would like you to set those numbers on your abacus. Each of you will have different numbers to set.

GROUP A:	8	22	20	24	41	47	29	18
GROUP B:	9	33	40	31	42	28	49	47
GROUP C:	7	44	30	42	23	36	38	29

This time I'm going to record eight numbers on your abacus. I would like you to read them to me. Each of you will have different numbers to read.

GROUP A:	6	44	23	28	35	31	30	37
GROUP B:	7	22	14	36	25	32	20	26
GROUP C:	8	11	34	16	39	42	40	17

LESSON FIVE*

1	It's time for another abacus lesson. You've already learned to set and clear the numbers from 0 to 49. In this lesson you'll learn to set and clear the numbers from 50 to 99.	NONE
2	When I call your name, use your index finger to set the upper bead in the ones column so that your abacus will say 5.	OBV
3	Now when I call on you, find the upper bead in the tens column and set it to the bar. It says five tens or fifty. Five tens and five or 55 should be recorded on your abacus.	OBV
4	Clear the 5 tens and 5 from your abacus.	OBV
5	When you use an upper bead and a lower bead which are in the same column to record a number, you should always move the lower bead first. If you wanted to record 6 on your abacus, would you set a lower bead or the five-bead first? No comment. or To record 6 you would set a lower bead and then the five-bead.	LOWER BEAD
6	Sixty is another name for six tens. Set 60 on your abacus.	OBV
7	Clear 60 from your abacus in the same order in which you set it.	OBV
8	This time set 64 on your abacus. Set 60 and then 4 ones. Clear 64 in the same order in which you set it.	OBV

9	Find the bead in the tens column which says 5 tens or 50 and set it to the bar. Then set the 4 and 5 of 9 in the ones column. If you set 59 correctly, the upper bead in the tens column should be set at the bar. All of the beads in the ones column should be set at the bar. Did you record 59 correctly? Clear 59 in the same order in which you set it.	YES
10	Set 66. To do so you will need to set 6 in the tens column and 6 in the ones column. Clear 66 in the same order in which you set it.	OBV
11	This time set 75. That means you will need to show 7 tens and 5. Clear 75 in the same order in which you set it.	OBV
12	Think 3+5=8 and set 8 on your abacus. Eighty means 8 tens. Add 80 to the 8 which is already set on your abacus. What number is recorded on your abacus? Clear 88.	88
13	Set 71. Before you clear 71, check to be sure that you have recorded it correctly. Seventy-one means 7 tens and 1, so you should have recorded 7 in the tens column and 1 in the ones column. Did you set 71 correctly? Clear 71.	YES

14	Set 8 tens and 6 or 86. Now, clear 86 in the same order in which you set it.	OBV
15	Since you've learned to use the tens column you've set 10, 20, 30, 40, 50, 60, 70 and 80 on the abacus. Now, set 9 tens or 90 on the abacus. Clear 90 by clearing the lower beads and then the upper bead.	OBV
16	You just set and cleared 90. This time set 97. Now, clear 97.	OBV

Criterion Check

I'm going to read eight numbers to you. I would like you to set those numbers on your abacus. Each of you will have different numbers to set.

GROUP A:	54	61	20	68	19	47	42	96
GROUP B:	51	72	10	97	16	28	31	87
GROUP C:	52	73	30	86	18	37	24	79

This time I'm going to record eight numbers on your abacus. I would like you to read them to me. Each of you will have different numbers to read.

GROUP A:	52	64	40	87	15	26	34	79
GROUP B:	53	84	30	89	17	39	21	69
GROUP C:	54	61	20	78	19	49	43	87

LESSON SIX*

	Let's begin this lesson with some questions that should help you review what you've already learned about the abacus.	
	1. Show me the finger that you should always use to set lower beads to the bar.	THUMB
	2. Show me the finger that is your tracking finger.	LEFT INDEX
1	3. To record the number 7 on an abacus you must use both upper and lower beads. Is it true that you should set the two lower beads before you set the five-bead?	YES
	4. Show me the finger you should always use to clear beads from the abacus.	RIGHT INDEX
	5. When you set a number like 43 on the abacus, you always set the digits in the same order in which you would write them. Should you clear beads in the same order in which you set them?	YES
	6. Show me the finger you should use to set an upper bead.	RIGHT INDEX
	Good. or Content restated affirmitively.	
2	Set 5 on your abacus. Now, find the tens column and set one lower bead to the bar. What number is recorded on your abacus?	15
3	Five plus ten is fifteen. When you set 5 and then set 10 were you adding or subtracting? Clear 15 from your abacus.	ADDING

4	Let's use the abacus to show that the sum of 25 and 4 is 29. First, set 25 or 2 tens and 5 on your abacus. Then, set 4 in the ones column. Is the number 29 recorded on your abacus? Clear the 2 tens and 9 of 29.	YES
5	Here's another problem. First, set 32 on your abacus. Then add 11 by setting 1 ten and 1. The correct answer is 43. Did you get the correct answer? Clear your abacus so you'll be ready for another problem.	32 +11 43 YES
6	Set 41 or 4 tens and 1 on your abacus. Add 52 to 41. Read your answer to me. I agree. or Your answer should be 93. Clear your abacus.	41 +52 93
7	This time begin by setting the number 58. To that number add 21. Read your answer silently to yourself. Seventy-nine is the correct answer. Use your right index finger to clear it.	58 +21 79
8	Let's add three numbers this time. Begin by setting 21 on your abacus. To 21 add the number 13. The number 34 or 3 tens and 4 should appear on your abacus Instead of clearing that number add 15. Read your answer to me. Good. or The correct answer is 49. Clear your abacus.	21 +13 34 +15 49

Criterion Check

So far I have read to you and explained exactly what you were to do. Now I want you to work some problems without any help from me. I will read the problems to you. If you would like me to read a problem more than once, quietly raise your hand. As soon as you have finished working a problem move your fingers so that I can see your answer. When everyone has finished the problem, I will ask someone to read the answer aloud. There are six problems in this group.

75	63	28	81	25	16
+24	+21	+51	+16	+51	+32
99	84	79	97	76	48
				+12	+50
				88	98

9	Set the number 28 on your abacus. Should you set or clear beads to show that 28-3=25?	CLEAR
10	Clear your abacus so we can work some subtraction problems. Set 98 or 9 tens and 8 on your abacus. From that number subtract 52. First clear 5 tens. Then clear 2 ones. Forty-six is the correct answer. Did you get the correct answer? Clear your abacus.	98 -52 46 YES
11	This time begin by setting 8 tens and 7 or 87 on your abacus. From that number subtract 25 by clearing 2 tens and 5. Read me your answer. Good. or The correct answer is 62. Clear your abacus.	87 -25 62
12	Set 99 on your abacus. Subtract 26 from that number. Your abacus should say 73, but instead of clearing it subtract 12. Read me your answer. I agree. or The correct answer is 61.	99 -26 73 -12 61

Criterion Check

It's time to work some more problems without any help from me. When you're finished working, you should move your hands away from your abacus so that I can see your answer. When everyone has finished, I'll ask one of you to read your answer aloud. There are six problems in this group.

$$\begin{array}{rrrrrr} 99 & 48 & 69 & 87 & 84 & 64 \\ -36 & -35 & -55 & -21 & -21 & -51 \\ \hline 63 & 13 & 14 & 66 & 63 & 13 \\ & & & & -53 & -10 \\ \cline{5-6} & & & & 10 & 3 \end{array}$$

You've begun to learn how to add on the abacus. You've also learned something about how to subtract. In this problem you'll do both.

13 Use your abacus to find the sum of 32 and 15. Now, instead of clearing 47, subtract 21.

$$\begin{array}{r} 32 \\ +15 \\ \hline 47 \\ -21 \\ \hline 26 \end{array}$$

Twenty-six is the correct answer. Did you get the correct answer?

YES

Clear your abacus.

14 Here's another problem that has both addition and subtraction in it. Begin by setting 64 on your abacus. To 64 add 25. Eighty-nine should be set on your abacus. Now subtract 53. Read your final answer to me.

$$\begin{array}{r} 64 \\ +25 \\ \hline 89 \\ -53 \\ \hline 36 \end{array}$$

Good. or Thirty-six is the correct answer.

Clear your abacus.

Criterion Check

It's time for some more problems which you should be able to work independently. Remember to move your hands as soon as you've finished each problem so that I can see your answer. As usual we will take turns reading the answers aloud. There are three problems in this group.

$$\begin{array}{rrr}
84 & 49 & 13 \\
\underline{-23} & \underline{-15} & \underline{+56} \\
61 & 34 & 69 \\
\underline{+33} & \underline{+55} & \underline{-14} \\
94 & 89 & 55
\end{array}$$

LESSON SEVEN*

1	It's time for another abacus lesson. So far you've learned to set and clear beads with your thumb and index finger. You've learned to record the numbers from 0 to 99 and have used them in addition and subtraction problems. In this lesson you'll learn to record the numbers from 100 to 999.	NONE
2	Set 23 on your abacus. To record that number you used two beads in the tens column and three in the ones column which is a way of saying that 23 means 2 tens and 3 ones.	OBV
3	Should the four of 400 be on the right or left of 23 if you recorded that number on an abacus? Raise your right or left hand to answer that question.	LEFT
4	Because numbers are shown on an abacus in the same order in which they are written the 400 should be set immediately to the left of 23. Set 400 so that your abacus will say 423.	OBV
5	To record 423 you set three beads in the ones column and two beads in the tens column. What is the column in which you set four beads called? No comment. or It is called the hundreds column.	HUNDREDS
6	Clear 23 from your abacus so that it will say exactly 400. The two columns where there are no beads set are like having zeros on paper.	OBV

7	Find the hundreds column on your abacus and clear two lower beads. What number is recorded on your abacus now? No comment. or The number 200 is recorded on your abacus. Clear your abacus.	200
8	As you move from right to left on an abacus each column is worth ten times as much as the one before it.	NONE
9	The upper bead in the ones column stands for 5. In the tens column the upper bead stands for 50. What number does the upper bead in the hundreds column stand for. No comment. or It stands for 500.	500
10	Set 500 on your abacus. Did you set with your index finger? Clear 500 with your index finger.	YES
11	Set 7. Remember to set the two lower beads before you set the five-bead. Now add 7 tens or 70 to 7. What number is recorded on your abacus?	77
12	Instead of clearing 77, add 700 so that your abacus will say 777. Clear 777. Clear in the hundreds column first.	OBV
13	This time set 148. First locate the hundreds column and set one bead to the bar. After you have set 100, move to the tens column and set 40. The final step is to set 8 in the ones column. Use your index finger to clear 148 in the same order in which you set it.	OBV

14	This time set 345. You should set each digit in the order in which you would write it. Clear the 3 hundreds, the 4 tens, and 5 of 345.	OBV
15	Should you use any beads in the tens column to set 804?	NO
16	Set 804. Now, clear 804 from your abacus.	OBV

Criterion Check

I'm going to read eight numbers to you. I would like you to set those numbers on your abacus. Each of you will have different numbers to set.

GROUP A: 14 68 432 206 765 654 987 324

GROUP B: 79 413 378 460 796 839 768 231

GROUP C: 24 87 212 409 657 985 678 421

This time I'm going to record eight numbers on the abacus. I would like you to read them to me. Each of you will have different numbers to read.

GROUP A: 43 57 321 309 987 725 697 323

GROUP B: 83 323 629 380 687 736 659 341

GROUP C: 32 65 422 802 568 683 788 211

LESSON EIGHT*

1	In an earlier lesson you learned how to add and subtract two-digit numbers on the abacus. Try this problem to warm up before we begin something new. Set 62 on your abacus. To that number add 15. Now instead of clearing your abacus subtract 27. Read me your answer. Good. or Fifty is the correct answer	62 +15 77 -27 50
2	This lesson also has addition and subtraction problems in it. However, it's a little different. The problems in this lesson have three-digit numbers in them. Clear your abacus so you will be ready for something new.	NONE
3	Set 135 on your abacus. To that number add 251. First, add 200 by setting two beads in the hundreds column. Then add 51 by setting 5 in the tens column and then 1. The correct answer is 386. Did you get the correct answer?	135 +251 386 YES
4	Begin in the hundreds column and clear 386. Show me the finger you used to clear your abacus. No comment. or You should clear with your right index finger.	RIGHT INDEX
5	Set 725 on your abacus. It has 7 hundreds, 2 tens, and 5. To that number add 152. Read me your answer. I agree. or Your answer should be 877. Clear your abacus.	725 +152 877

6 This time find the sum of 523 and 451. First set 523. Then add 4 hundreds, 5 tens and 1. Read me your answer.

Fine. or Your answer should be 974.

Clear your abacus so you'll be ready for another problem.

$$\begin{array}{r} 523 \\ +451 \\ \hline 974 \end{array}$$

---

7 This time let's add three numbers. Begin by setting 512. Now, add 251. Instead of clearing 763 add 121.

Your final answer should be 884. Did you get the correct answer?

Clear your abacus.

$$\begin{array}{r} 512 \\ +251 \\ \hline 763 \\ +121 \\ \hline 884 \end{array}$$

YES

Criterion Check

I'm going to read six problems which you should be able to work independently. I'll ask one of you to read your answer and tell you whether that answer was correct. If you would like me to repeat a problem quietly raise your hand.

$$\begin{array}{r} 241 \\ +555 \\ \hline 796 \end{array} \qquad \begin{array}{r} 356 \\ +540 \\ \hline 896 \end{array} \qquad \begin{array}{r} 186 \\ +213 \\ \hline 399 \end{array} \qquad \begin{array}{r} 381 \\ +512 \\ \hline 893 \end{array} \qquad \begin{array}{r} 325 \\ +123 \\ \hline 448 \\ +551 \\ \hline 999 \end{array} \qquad \begin{array}{r} 552 \\ +235 \\ \hline 787 \\ +102 \\ \hline 889 \end{array}$$

---

8 Now, let's do some subtraction problems. Set 934 on your abacus. From that number subtract 222.

The correct answer is 712. Did you get the correct answer?

Use your index finger to clear your abacus.

$$\begin{array}{r} 934 \\ -222 \\ \hline 712 \end{array}$$

YES

---

9 This time begin by setting 447 on your abacus. From that number subtract 145. Read me your answer.

Good. or Your answer should be 302.

Clear your abacus.

$$\begin{array}{r} 447 \\ -145 \\ \hline 302 \end{array}$$

10 This time let's try a subtraction problem that has three numbers in it. Set 897 on your abacus. From that number subtract 525. Now instead of clearing your abacus subtract 121. Read me your answer

897
-525
372
-121
251

Good. or Your final answer should be 251.

Clear your abacus.

Criterion Check

I'm going to read six problems which you should be able to work independently. I'll ask one of you to read your answer and tell you whether that answer was correct. If you would like me to repeat a problem quietly raise your hand.

789	939	878	438	949	879
-135	-511	-327	-225	-234	-152
654	428	551	213	715	727
				-510	- 25
				205	702

11 This time let's do a problem that has both addition and subtraction in it. Set 486 on your abacus. From that number subtract 251. Check to see that 235 is recorded on your abacus. Then add 514. Read me your final answer.

486
-251
235
+514
749

Your final answer should be 749. Begin in the hundreds column and clear your abacus.

12 Here's another problem that has both addition and subtraction in it. Set 726 on your abacus. To that number add 63. Now instead of clearing your abacus subtract 578.

726
+ 63
789
-578
211

211 is the correct answer. Was your answer correct?

YES

You may clear your abacus.

Criterion Check

I'm going to read three problems. After you've finished I'll ask you to read your answer aloud. If you'd like me to repeat a problem I will.

$$\begin{array}{r} 683 \\ +115 \\ \hline 798 \\ -543 \\ \hline 255 \end{array} \qquad\qquad \begin{array}{r} 946 \\ +\ 52 \\ \hline 998 \\ -841 \\ \hline 157 \end{array} \qquad\qquad \begin{array}{r} 478 \\ -222 \\ \hline 256 \\ +523 \\ \hline 779 \end{array}$$

LESSON NINE*

1	So far you've learned to record three digit numbers and to add and subtract with them on the abacus. In this lesson you'll learn to use the thousands column.	NONE
2	The ones column is easy to find because it is next to the right-hand edge of your abacus. The tens column is just to the left of the ones column and the hundreds column is immediately to the left of the tens column. Which side of the hundreds column is the thousands column on? No comment. or The thousands column is to the left of the hundreds column.	LEFT
3	In the ones column the upper bead stands for 5. In the tens column the upper bead stands for 50 and in the hundreds column it stands for 500. What does the upper bead in the thousands column stand for? No comment. or It stands for 5000.	5000
4	Can we say that as you move from right to left on the abacus each column is worth ten times as much as the one before it? No comment. or Yes, we could say that.	YES
5	Set 51. Now add 200 so that your abacus will say 251. Use your tracking finger to locate the thousands column and add 5000 so that your abacus will say 5251. In what column should you begin to clear 5251? Do it. or Begin in the thousands column and clear your abacus.	THOUSANDS

Will you need to set a bead in the hundreds column to record 4,079?

6 No comment. or You will not need a bead in the hundreds column to record 4,079 on your abacus. NO

Now, clear it using your index finger.

Set 86. Now, set 2 beads in the hundreds column. Finally, set 7 in the thousands column.

7 What number is recorded on your abacus? 7286

Clear 7,286. of The number 7,286 should be recorded on your abacus. Clear your abacus.

Criterion Check

I'm going to read eight numbers to you. I would like you to set those numbers on your abacus. Each of you will have different numbers to set.

GROUP A: 845 550 9,145 3,208 6,759
6,406 7,592 1,416

GROUP B: 736 570 8,236 4,107 7,685
7,039 8,265 9,243

GROUP C: 936 505 8,245 4,190 5,678
7,280 9,763 2,237

This time I'm going to record eight numbers on the abacus. I would like you to read those numbers to me. Each of you will have different numbers to read.

GROUP A: 718 160 6,524 9,083 9,876
7,021 8,753 2,347

GROUP B: 629 280 5,731 8,072 6,897
6,840 7,185 3,115

GROUP C: 827 207 7,425 8,094 6,789
7,309 6,892 1,239

8	You've already learned to add and subtract with numbers which have two and three digits. Try this problem. What is the sum of 224 and 505? Read me your answer, but do not clear your abacus. I agree. or I disagree. Your answer should be 729.	224 +505 729
9	The number 729 should be recorded on your abacus. From that number subtract 116. Read me your final answer. I agree. or Your final answer should be 613. Clear your abacus to be ready for something new.	729 -116 613
10	Now let's work some problems that have four digits in them. That means that you will add and subtract with numbers as large as 9999.	NONE
11	To begin set 2 thousands, 5 hundreds, 4 tens and 1 or 2541 on your abacus. To that number add 5358. These are the largest numbers you've ever had in an abacus problem so remember to look carefully above and below the bar when reading your answer. The correct answer is 7899. Did you get the correct answer? You should begin in the thousands column when you clear your abacus.	2541 +5358 7899 YES
12	Here's another addition problem. Set 3,762 on your abacus. To that number add 5,122. Read me your answer. I agree. or Your final answer should be 8,884.	3762 +5122 8884

13 Clear 8,884 from your abacus. What column did you clear first?

THOUSANDS

Good. or You should have cleared in the thousands column first.

Criterion Check

I'm going to read four problems that you should be able to work independently. After you've finished I'll ask you to read your answer aloud.

4381	7365	2536	1416
+5513	+2523	+1352	+3552
9894	9888	3888	4968

Now that you've worked several problems in which you add two four-digit numbers, let's work one in which you add three four-digit numbers.

14 Set 2,468 on your abacus. To that number add 1,510. Your abacus should say 3,978. Instead of clearing your abacus add 5,021.

2468
+1510
3978
+5021
8999

The correct answer is 8,999. Did you get the correct answer?

Clear your abacus.

Here's another problem in which you will add three four-digit numbers. First set 3,516. To that number add 1,231. Now, instead of clearing 4,747 add 5,252. Read me your answer.

3516
+1231
4747
+5252
9999

15 Good. or Your final answer should be 9,999.

Begin in the thousands column and use your right index finger to clear your abacus.

Criterion Check

I'm going to read four problems which you should be able to work. After you've both finished I'll ask one of you to read your answer aloud.

$$\begin{array}{rrrr} 3315 & 1234 & 5172 & 1332 \\ +5172 & +2115 & +1615 & +2515 \\ \hline 8487 & 3349 & 6787 & 3847 \\ +\ 512 & +5550 & +2112 & +\ 150 \\ \hline 8999 & 8899 & 8899 & 3997 \end{array}$$

Now let's try a subtraction problem that
has four-digit numbers in it. To begin
set 8,639 on your abacus. From that num-
ber subtract 2,527. Be sure you look
above and below the bar when you read
16 your answer.

$$\begin{array}{r} 8639 \\ -2527 \\ \hline 6112 \end{array}$$

I agree. or Your final answer should be 6,112.

Clear your abacus.

Here's another subtraction problem. Begin by setting 8,948 on your abacus. From that number subtract 7,635. Read me your answer.

17

$$\begin{array}{r} 8948 \\ -7635 \\ \hline 1313 \end{array}$$

I agree. or Your final answer should be 1,313.

Clear your abacus.

Criterion Check

I'm going to read four problems which you should be able to work independently. After you've both finished I'll ask one of you to read your answer aloud. If you would like me to repeat a problem, I will.

$$\begin{array}{rrrr} 4899 & 8934 & 9899 & 7384 \\ -2635 & -5631 & -3567 & -7263 \\ \hline 2264 & 3303 & 6332 & 121 \end{array}$$

Now that you've worked both addition and subtraction problems let's try a problem that has both kinds of calculation in it.

18 To begin set 4,281 on your abacus. To that number add 5,111. Your abacus should say 9,392 but instead of clearing it subtract 3,251.

The correct answer is 6,141. Did you get the correct answer?

You may clear your abacus.

4281
+5111
9392
-3251
6141

YES

---

19 Here's another problem that has both addition and subtraction in it. Set 1,898 on your abacus. From that number subtract 262. Now add 2,351. Read me your answer.

Good. or Your final answer should be 3,987.

You may clear your abacus.

1898
- 262
1636
+2351
3987

Criterion Check

I'm going to read three problems which you should be able to work. After you've finished I'll ask one of you to read your answer aloud. If you'd like me to repeat a problem I will.

6422	7865	4357
- 210	+2124	+5631
6212	9989	9988
+2635	-3567	-4567
8847	6422	5421

LESSON TEN*

1	You've begun to learn how to add and subtract on the abacus. Set 9 on your abacus. If I asked you to add 6 could you set a one-bead and a five-bead to the bar?	NO
2	You know that 9+6=15, but I don't expect you to be able to show it on your abacus. In this lesson you will begin to learn some rules that will help you add when there are not enough beads to set at the bar. Clear your abacus so that you will be ready for something new.	NONE
3	Set 4 on your abacus to show that a clerk has four cents in his cash register.	OBV
4	Now, set 5 to show that you gave the clerk a nickel to pay for a penny piece of bubble gum.	OBV
5	The clerk would give you four cents change so clear 4 from your abacus.	OBV
6	Your abacus should show how much money the clerk would have now. What number is recorded there? Good. or The number 5 should be recorded on your abacus. Clear your abacus.	5
7	By clearing four lower beads and setting the five-bead, you used your abacus to show that 4+1=5.	NONE

8	Set the number 14 on your abacus. Is there another lower bead in the ones column which you can use to show that 14+1=15?	NO
9	Because there is not another lower bead which you can use to add 1, you should add one by clearing 4 and setting 5. Do it. Clear the 1 ten and 5 of 15 from your abacus.	14 + 1 15 OBV
10	Set 54 on your abacus. Were all of the beads which stand for one used to record 54?	YES
11	Use your index finger to clear 4 and set 5 as a way of adding 1. Is the number 55 recorded on your abacus? Clear your abacus.	YES
12	As we continue to work problems on the abacus there will be many times when it is impossible to add by setting a bead directly to the bar. You have just learned a way of adding 1 indirectly. Over time you'll learn several other rules to help you add on the abacus.	NONE
13	I'm going to read you an addition problem. Even though you could find the answer in your head use your abacus to help you. Look for a chance to use the rule Clear 4, set 5 as a way of adding 1. The problem is 2+2+1+3. Your final answer should be 8. You may clear your abacus.	OBV
14	How can you add one when it is not possible to set a one-bead directly to the bar? I agree. or When it is not possible to set a bead to the bar you can add one by clearing 4 and setting 5.	CLEAR 4, SET 5

15 Set 45 on your abacus. Suppose you wanted to add 10 to 45. In which column would you add 1 by clearing 4 and setting 5? TENS

No comment. or You would need to clear 4 and set 5 in the tens column.

---

16 Add 10 to 45 and read me your answer. OBV

Fifty-five is the correct answer. You may clear your abacus.

Criterion Check

I'm going to read eight problems which you should be able to work. After you've finished I'll ask you to read your answer aloud. If you'd like me to repeat a problem raise your hand.

41	14	42	84	46	34	42	2
+15	+51	+12	+11	+13	+51	+12	+2
56	65	54	95	59	85	1	1
						4	12
						59	31
							11
							59

---

17 Now that you've worked several problems with two-digit numbers, let's work a problem that has three-digit numbers in it. In this problem you'll need to use the rule for adding 1 in the hundreds column. Set 423 on your abacus. To that number add 165.

423
+165
588

588 is the correct answer. Did you get the correct answer?

YES

Clear your abacus.

18	Set the number 462 on your abacus. To that number add 1 hundred, 1 ten, and 1 or 111. As you work think about whether you are using the rule for adding 1 in the hundreds column, the tens column or the ones column. In what column did you apply the rule for adding 1? No comment. or You should have used the rule in the hundreds column. Clear your abacus.	HUNDREDS
19	A rule for adding on the abacus can be applied in any column. In this problem you will need to use the rule for adding 1 in the thousands column. Set 4,264 on your abacus. To that number add 1,235. Read me your answer. Begin in the thousands column when you clear 5,499.	4264 +1235 5499
20	To work this problem which has four-digit numbers in it you will need to use the rule for adding 1 more than one time. To begin set 2,464 on your abacus. To that number add 2,131. The correct answer is 4,595. Did you get the correct answer? Clear your abacus.	2464 +2131 4595 YES

Criterion Check

I'm going to read seven problems which you should be able to work independently. After you've both finished, I'll ask one of you to read your answer aloud. If you would like me to repeat a problem, I will.

455	4321	524	2456	463	6342	415
+104	+1521	+121	+2122	+125	+2115	+132
559	5842	645	4578	588	8457	547
		+ 13	+1320			+ 12
		658	5898			559

LESSON ELEVEN

In the last lesson you learned that it was possible to add 1 by clearing 4 and setting 5. Before we begin something new, let's work a problem in which you'll use that way of adding 1.

1	Set 24 on your abacus. To that number add 21. Now, instead of clearing your abacus add 13. Read me your answer. Good. or Your final answer should be 58. Clear your abacus.	24 +21 45 +13 58
2	The rule clear 4, set 5 is one of several rules which explain how to add on the abacus. In this lesson you'll learn how to show on your abacus that 3+2=5.	NONE
3	Set 3 on your abacus. Are there 2 more beads below the bar which you could use to show that 3+2=5.	NO
4	Make your abacus say 5.	OBV
5	To record 5 you cleared three lower beads and set the five-bead. Can we say that when you cleared 3 and set 5 you added 2? Clear 5.	YES
6	In each column on your abacus there are beads below the bar with which you can count to 4 and a bead above the bar which stands for 5. Sometimes there are enough beads so that you can add by setting beads directly to the bar. When there are not enough beads available you must use a special rule to help you. Before you can decide whether or not to use a special rule you must check to see how many beads are available.	NONE

7	Set 4 on your abacus. Will you need a special rule to show that 4+2 is 6? No comment. or Yes you will.	YES
8	To clear 3 and set 5 is a way of adding 2. Try that rule to see if it will help you show that 4+2=6. Did the rule work?	YES
9	Hold up the finger you used to clear 3 and set 5. Good. or You should have used your right index finger. Clear 6 from your abacus.	RIGHT INDEX
10	Here's a problem in which you'll need to use the rule clear 3, set 5 to add 2. Set 23 on your abacus. To that number add 52. The correct answer is 75. Was your answer correct? Clear your abacus. Remember to clear the 7 tens before you clear the 5.	23 +52 75 YES
11	This time begin by setting 31 on your abacus. To that number add 2 tens and 5 or 25. You'll need to use the rule clear 3, set 5 in the tens column. Read me your answer. Good. or Fifty-six is the correct answer. Clear your abacus.	31 +25 56
12	When all four lower beads are set to the bar, you can add 2 by clearing 3 and setting 5. Try it in this problem. Set 44 on your abacus. To that number add 22. Your answer should be 66. Did you get the correct answer? Clear your abacus.	44 +22 66 YES

13 Try this problem. Remember that you may add 2 by clearing 3 and setting 5. Set 34 on your abacus. Now add 2 tens and 2 or 22. Read me your answer.

$$\begin{array}{r} 34 \\ +22 \\ \hline 56 \end{array}$$

Good. or Fifty-six is the correct answer.

Clear your abacus.

Criterion Check

I'm going to read four problems which you should be able to work. After you've finished I'll ask you to read your answer aloud. If you'd like me to repeat a problem raise your hand.

$$\begin{array}{r} 43 \\ +52 \\ \hline 95 \end{array} \qquad \begin{array}{r} 31 \\ +26 \\ \hline 57 \end{array} \qquad \begin{array}{r} 24 \\ +22 \\ \hline 46 \end{array} \qquad \begin{array}{r} 43 \\ +26 \\ \hline 69 \end{array}$$

---

14 You've learned to add 2 by clearing 3 and setting 5. At the top of the page which I just gave you are three problems. The correct answers have been given. Circle the one problem in which the special rule for adding 2 was used. You should use your abacus to help you decide on the best answer.

ANSWER STARRED

38 + 51 = 89
15 + 23 = 38
*53 + 42 = 95

You should have marked the third problem.

---

15 There are also three problems on the bottom half of the page. The correct answer have been given. This time circle the one problem in which you would NOT need to use the rule for adding 2. You should use your abacus to help you decide on the best answer.

ANSWER STARRED

53 + 32 = 85
*66 + 12 = 78
41 + 25 = 66

You should have marked the second problem.

16	I'm going to read three phrases which explain how to move beads on the abacus. When I have finished reading, set 1, 2, or 3 on your abacus to show whether the first, second, or third phrase explains a way of adding 1. Set 2 Clear 3, set 5 *Clear 4, set 5 To clear 4 and set 5 is a way of adding 1.	ANSWER STARRED
17	Set 44 on your abacus. To that number add 51. As you work think about whether you use a rule for adding 1 or a rule for adding 2. To find the sum 95 did you use a rule for adding 1 or a rule for adding 2? No comment. or You should have used a rule for adding 1 to help you. Clear your abacus.	$\begin{array}{r}44\\+51\\\hline 95\end{array}$
18	In this problem you will need to use the rule for adding 1 and the rule for adding 2. Set 42 on your abacus. To that number add 12. Now, instead of clearing 54 add 42. Read me your answer. Good. or The correct answer is 96. You may clear your abacus.	$\begin{array}{r}42\\+12\\\hline 54\\+42\\\hline 96\end{array}$

Criterion Check

I'm going to read four problems which you should be able to work. After you've finished I'd like you to read your answer aloud. If you would like I will repeat a problem.

$$\begin{array}{r}24\\+22\\\hline 46\\+13\\\hline 59\end{array}\qquad\begin{array}{r}32\\+22\\\hline 54\\+41\\\hline 95\end{array}\qquad\begin{array}{r}34\\+21\\\hline 55\end{array}\qquad\begin{array}{r}44\\+12\\\hline 56\end{array}$$

This time let's try a problem that has
three-digit numbers in it. Think care-
fully as you work. Set 452 on your abacus.
To that number add 232. Be sure you look
19 above and below the bar as you read your
answer.

$$\begin{array}{r}452\\+232\\\hline 684\end{array}$$

Good. or Your answer should be 684.

Use the index finger of your right hand to clear your abacus.

This problem has both addition and sub-
traction in it. You'll need to use the
rule for adding 1 and the rule for add-
ing 2. First, set 879. From that number
subtract 525. Instead of clearing 354 add
20 231.

$$\begin{array}{r}879\\-525\\\hline 354\\+231\\\hline 585\end{array}$$

Your final answer should be 585. Did you get the correct answer.

YES

Clear your abacus.

Criterion Check

I'm going to read four problems which you should be able to work. After you've finished I'll ask you to read your answer aloud. If you'd like me to repeat a problem raise your hand.

$$\begin{array}{rrrr}463 & 443 & 544 & 844\\+132 & +215 & +312 & +112\\\hline 595 & 658 & 856 & 956\\ & & -251 & \\\cline{3-3} & & 605 & \end{array}$$

This addition problem has four-digit numbers in it. First, set 4264. To that number add 1,632.

$$\begin{array}{r}4264\\+1632\\\hline 5896\end{array}$$

21 The correct answer is 5,896. Was your answer correct?

YES

Begin in the thousands column when you clear your abacus.

This time let's work a problem in which all the numbers have either three or four digits. Set 132 on your abacus. To that number add 4,322. Now, instead of clearing 4,454, add 1,235. Look above and below the bar and read your answer.

$$\begin{array}{r} 132 \\ +4322 \\ \hline 4454 \\ +1235 \\ \hline 5689 \end{array}$$

I agree. or The correct answer is 5,689.

Clear your abacus.

Criterion Check

I'm going to read three problems which you should be able to work independently. After you've finished I'll ask you to read your answer aloud. Raise your hand if you'd like me to repeat a problem.

$$\begin{array}{r} 6484 \\ +3211 \\ \hline 9695 \end{array} \qquad \begin{array}{r} 3241 \\ +2516 \\ \hline 5757 \end{array} \qquad \begin{array}{r} 3456 \\ +\ 132 \\ \hline 3588 \\ +2311 \\ \hline 5899 \end{array}$$

LESSON TWELVE*

1	Before beginning something new, let's work a problem that is like some of the ones you've already worked. You'll need to use both of the rules for addition which you've already learned. Set 44 on your abacus. To that number add 21. Read me your answer. I agree. or I disagree your answer should be 65. Clear your abacus so that you'll be ready for something new.	$\begin{array}{r}44\\+21\\\hline 65\end{array}$
2	Learning the rules of abacus calculation will help you to add quickly and accurately on the abacus. These lessons are written so that you will learn one new rule in each lesson. You've already learned a way of adding 1 and 2 on the abacus. In this lesson you'll learn a rule for adding 3. Set 2 on your abacus. How many lower beads are there which you haven't used?	2
3	Can you show that 2+3=5 by using only the lower beads?	NO
4	You already know that 2+3=5. Make your abacus say 5.	OBV
5	To record 5 on your abacus you cleared 2 lower beads and set the five-bead to the bar. Can we say that clearing 2 and setting 5 is a way of adding 3? No comment. or To clear 2 and set 5 is a way of adding 3. Clear 5.	YES

6	Here's a problem in which you should add 3 by clearing 2 and setting 5. Set 42 on your abacus. To that number add 53. 95 is the correct answer. Did you get the correct answer? Clear your abacus.	42 +53 95 YES
7	To clear 2 and set 5 is a way of adding 3. Show me the finger which you should use to clear 2 and set 5. No comment. or You should use your right index finger.	RIGHT INDEX
8	Set 3 on your abacus. Will you need to clear 2 and set 5 to show that 3+3=6? Try it. Now, clear 6 from your abacus.	YES
9	Try this problem. Set 33 on your abacus. To that number add 35. Remember to clear 2 and set 5 when you add the 3 tens of 30. Read me your answer. Good. or Sixty-eight is the correct answer. You may clear your abacus.	33 +35 68
10	Set 4 on your abacus. Are there any more beads below the bar which you can use to show that the sum of 4+3=7?	NO
11	You've already used the rule clear 2, set 5 to show that 2+3=5 and to show that 3+3=6. Try that rule to see if it will help you show that 4+3=7. Did the rule work? Clear your abacus.	YES

12	Set 5 on your abacus. Suppose you wanted to show that 5+3=8. Would you need to use the rule clear 2, set 5 to add 3?	NO
13	Because there were three lower beads that you could set to the bar, you would not need to use a special rule. It's important to know when and how to use each special rule. Clear your abacus so you'll be ready for something new.	NONE
14	I've given you a page that has three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used the rule clear 2, set 5 to add 3. *28 + 31 = 59 *23 + 23 = 46 14 + 35 = 49 You should have circled the first two problems.	ANSWER STARRED
15	This time let's add three numbers. You'll need to use the rule for adding three more than once. Set 41 on your abacus. To that number add 33. Now, instead of clearing 74 add 2 tens and 3 or 23. Your final answer should be 97. Did you get the correct answer? Clear your abacus.	41 +33 74 +23 97 YES

Criterion Check

I'm going to read eight problems which you should be able to work independently. After you've finished I'll ask you to read your answer aloud. If you'd like me to repeat a problem raise your hand.

47	32	42	22	24	21	13	44
+31	+31	+53	+31	+23	+33	+13	+30
78	63	95	53	47	54	26	74
			+33				+13
			86				87

16	Now, let's work some problems with numbers that have more than two digits. You'll have to decide when to use the rule for adding 3. Set 325 on your abacus. To that number add 533. The correct number is 858. Did you get the correct answer? Clear your abacus.	325 +533 858 YES
17	In this problem you'll need to use the rule for adding 3 and one of the rules you've already learned. Set 423 on your abacus. To that number add 2 hundreds, 5 tens and 3 or 253. Be sure you look above and below the bar when you read me your answer. Good. or Your answer should be 676. Clear your abacus.	423 +253 676
18	This problem has both addition and subtraction in it. Begin by setting 34 on your abacus. To that number add 31. Now, instead of clearing 65 subtract 15. Read me your answer. Good. or Fifty is the correct answer. Use your index finger to clear your abacus.	34 +31 65 -15 50
19	Here's another problem that has both addition and subtraction in it. Set 304 on your abacus. To that number add 381. Now, instead of clearing 685 subtract 510. Read me your answer. I agree. or The correct answer is 175. Clear your abacus.	304 +381 685 -510 175

20	This time let's begin by setting a very large number and then subtract from it. Set 9,798. From that number subtract 3,555. Now, instead of clearing 6,243, add 2,313. Your final answer should be 8,556. Did you get the correct answer? Clear your abacus.	9798 -3555 6243 +2313 8556 YES

Criterion Check

I'm going to read seven problems which you should be able to work. When you've finished, I'll ask you to read your answer aloud. Raise your hand if you'd like me to repeat a problem.

215	423	43	445	7445	644	40
+334	+153	+11	+324	+1234	+332	+22
549	576	54	769	8679	976	62
		+13	-102	-2016		+23
		67	667	6663		85

LESSON THIRTEEN*

1	So far you've learned rules that will help you add 1, 2, and 3 on the abacus. In this lesson you'll learn a rule that will help you add 4.	NONE
2	Before we begin something new, let's work a problem in which you can use what you've learned so far. Here's the problem. Set 243 on your abacus. To that number add 212. Finally, instead of clearing your abacus add 321. Read me your answer. I agree. or Your answer should be 776. Clear your abacus.	243 +212 455 +321 776
3	Set 1 on your abacus as if you were a clerk with only one penny in the cash register.	OBV
4	Now, set 5 to show that a customer used a nickel to pay for four cents worth of candy.	OBV
5	The clerk would give the customer a penny in change so clear 1 from you abacus.	OBV
6	You were adding when you cleared 1 and set 5. What number did you add? Clear 4.	4
7	In this problem you'll need to use the rule clear 1, set 5 to add 4. Set 21 on your abacus. To that number add 5 tens and 4 or 54. 75 is the correct answer. Did you get the correct answer? Clear your abacus.	21 +54 75 YES

8 Here's another problem in which you can use that way of adding 4. Set 15 on your abacus. To that number add 4 tens and 2 or 42. Read me your answer.

$$\begin{array}{r}15\\+42\\\hline 57\end{array}$$

Good. or Fifty-seven is the correct answer.

Clear your abacus.

9 Set 2 on your abacus. Can you se the rule clear 1, set 5 to show that 2+4=6? Try it before you answer.

No comment. or You should be able to show that 2+4=6 by clearing 1 and setting 5.

YES

Clear your abacus.

10 Set 3 on your abacus. Will you need to use the rule clear 1, set 5 to show that 3+4=7? Do it.

YES

Clear your abacus.

11 Here's a problem in which you'll need to use the rule for adding 4 more than once. Set 32 on your abacus. To that number add 4 tens and 4 or 44. Look above and below the bar and read me your answer.

$$\begin{array}{r}32\\+44\\\hline 76\end{array}$$

Good. or The correct answer is 76.

Clear your abacus.

12 This time find the sum of 14 and 24. Look for a chance to use the rule for adding 4. Read me your answer.

$$\begin{array}{r}14\\+24\\\hline 38\end{array}$$

Fine. or Thirty-eight is the correct answer.

You may clear your abacus.

Criterion Check

I'm going to read four problems which you should be able to work independently. I'll ask one of you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{r}41\\+54\\\hline 95\end{array}\qquad\begin{array}{r}23\\+45\\\hline 68\end{array}\qquad\begin{array}{r}61\\+34\\\hline 95\end{array}\qquad\begin{array}{r}324\\+404\\\hline 728\end{array}$$

13 At the top of the page I have given you three problems. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used the rule for adding 4.

ANSWERS STARRED

*31 + 54 = 85
*22 + 46 = 68
*44 + 44 = 88

You should have marked all three problems.

14 At the bottom of the page I have given you are four phrases which explain how to add on the abacus. After each phrase are the numbers 1, 2, 3, and 4. Read each phrase and then circle the number that can be added using that rule.

ANSWERS UNDERLINED

Clear 2, Set 5   1 2 $\underline{3}$ 4
Clear 1, Set 5   1 2 3 $\underline{4}$
Clear 3, Set 5   1 $\underline{2}$ 3 4
Clear 4, Set 5 $\underline{1}$ 2 3 4

Restatement of each rule indicating the number that should have been marked.

15 In this problem you'll need to use more than one rule. Set 32 on your abacus. To that number add 34. Read me your answer.

$$\begin{array}{r}32\\+34\\\hline 66\end{array}$$

Good. or Your final answer should be 66.

Begin in the tens column and use your index finger when you clear your abacus.

16 This time let's add three numbers in succession. Set 32 on your abacus. To that number add 22. Now, instead of clearing your abacus add 41. Read me your answer.

Good. or The Correct answer is 95.

Clear your abacus.

$$\begin{array}{r} 32 \\ +22 \\ \hline 54 \\ +41 \\ \hline 95 \end{array}$$

Criterion Check

I'm going to read four problems which you should be able to work independently. I'll ask one of you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{rrrr} 22 & 31 & 24 & 14 \\ +23 & +22 & +43 & +43 \\ \hline 45 & 53 & 67 & 57 \\ +43 & +24 & & \\ \cline{1-2} 88 & 77 & & \end{array}$$

---

17 Try this problem which has three-digit numbers in it. Set 344 on your abacus. To that number add 231. Look above and below the bar when you read your answer.

Good. or Your answer should be 575.

Clear your abacus.

$$\begin{array}{r} 344 \\ +231 \\ \hline 575 \end{array}$$

---

18 This time begin by setting 442 on your abacus. To that number add 425. Read me your answer.

I agree. or The correct answer is 867.

Clear your abacus.

$$\begin{array}{r} 442 \\ +425 \\ \hline 867 \end{array}$$

---

19 This problem has both addition and subtraction in it. Set 749 on your abacus. From that number subtract 526. Your abacus should say 223. Instead of clearing it add 324. Read me your answer.

Good. or Your final answer should be 547.

Clear your abacus.

$$\begin{array}{r} 749 \\ -526 \\ \hline 223 \\ +324 \\ \hline 547 \end{array}$$

Criterion Check

I'm going to read four problems which you should be able to work independently. I'll ask one of you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{r} 451\\ +144\\ \hline 595 \end{array} \qquad \begin{array}{r} 124\\ +242\\ \hline 366 \end{array} \qquad \begin{array}{r} 113\\ +433\\ \hline 546\\ -525\\ \hline 21 \end{array} \qquad \begin{array}{r} 822\\ +143\\ \hline 965 \end{array}$$

This time let's work a problem that has four-digit numbers in it. Set 3,153 on your abacus. To that number add 3 thousands, 4 hundreds, 3 tens, and 6 or 3,436. Read me your answer.

20 I agree. or The correct answer is 6,589.

$$\begin{array}{r} 3153\\ +3436\\ \hline 6589 \end{array}$$

Clear your abacus so you'll be ready for another problem.

Begin this problem by setting 9,784. From that large number subtract 6,582. Now instead of clearing your abacus, add 3,456. Read me your answer.

21 Good. or The correct answer is 6,658.

$$\begin{array}{r} 9784\\ -6582\\ \hline 3202\\ +3456\\ \hline 6658 \end{array}$$

Clear your abacus.

Criterion Check

I'm going to read four problems which you should be able to work independently. I'll ask one of you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{r} 2345\\ +4514\\ \hline 6859 \end{array} \qquad \begin{array}{r} 4444\\ +4231\\ \hline 8675 \end{array} \qquad \begin{array}{r} 9964\\ -5713\\ \hline 4251\\ +4325\\ \hline 8576 \end{array}$$

LESSON FOURTEEN*

1	So far you've learned a way of adding 1, 2, 3, and 4. What number do you think you'll learn how to add in this lesson?	5
2	If I began counting or adding by 5's I would say 5, 10, 15, 20, 25. What number should I say next?	30
3	Set 30 on your abacus.	OBV
4	Add 5 to the 30 which is recorded on your abacus. Remember to set the five-bead with your right index finger. Did you need a special rule to show that the sum of 30 plus 5 is 35?	NO
5	Thirty-five plus five is 40. Think for a minute and make your abacus say 40.	OBV
6	To show 40 on your abacus you cleared the five-bead. What did you do with the ten bead?	SET IT
7	Can we say that clearing 5 and setting 10 is a way of adding 5? Clear your abacus.	YES
8	Is the bead which says ten to the right or left of the bead which says five? Raise your right or left hand to answer that question.	LEFT
9	Because the ten-bead is to the left of the five-bead, we can say that you cleared 5 and set one left.	NONE

10 When you add 5 by clearing 5 and setting one left, you should clear the five-bead with your right index finger. Show me the finger you should use to set the ten-bead? RIGHT THUMB

No comment. or You should set the ten-bead with your thumb.

---

11 A good way to get the feel of what it is like to use the rule for adding 5 is to use your abacus for counting 5's just as we were doing at the beginning of the lesson. Set 5 on your abacus. OBV

---

12 I'm going to count slowly from 5 to 45 by 5's. You should add on your abacus as I count. Some of the time you'll need to use the rule clear 5, set 1 left. 5, 10, 15, 20, 25, 30, 35, 40, 45. OBV

Clear your abacus.

Criterion Check

I'm going to read four problems which you should be able to work independently. After you've finished I'll ask you to read your answer aloud. Raise your hand if you'd like me to repeat a problem.

25	20	35	25
+15	+ 5	+55	+ 5
40	15	90	14
	40		44

---

13 Set 6 on your abacus. Try the rule clear 5, set 1 left to see if it will help you show that the sum of 6 plus 5 is 11. Did the rule work?

No comment. or The rule should have worked. YES

Clear your abacus.

14 The rule clear 5, set 1 left is a way of adding 5 when it is not possible to set a five-bead directly to the bar. I've given you a sheet which has three simple problems on it. Circle the one problem in which you would not need to use the rule clear 5, set 1 left to show the sum on your abacus. ANSWER STARRED

7 + 5 = 12
8 + 5 = 13
*3 + 5 = 8

You should have marked the third problem.

---

15 Here's another problem in which you'll need to use the rule clear 5, set 1 left to add 5. Set 27 on your abacus. To that number add 1 ten and 5 or 15. Read me your answer.

$$\begin{array}{r} 27 \\ +15 \\ \hline 42 \end{array}$$

Good. or The correct answer is 42.

Clear your abacus.

---

16 Begin by setting 4 on your abacus. Add 5 so that the number 9 will be recorded on your abacus. Instead of clearing your abacus continue to add by 5's until either 41 or 49 is recorded on your abacus. Some of the time you'll need to use the rule clear 5, set 1 left to help you add 5.

If you added correctly the number 49 should appear on your abacus. Did you add correctly? YES
Clear your abacus.

---

17 I have given you a sheet with three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used the rule for adding 5. You should use your abacus to help you decide on the best answer. ANSWER STARRED

*26 + 65 = 91
*17 + 55 = 72
*29 + 65 = 94

You should have used the rule clear 5, set 1 left to work all three problems.

18 You've learned that as you move to the left on an abacus each column is worth ten times as much as the one before it. Set 50 on your abacus. Now, add 5 tens or 50 to the fifty that is already set on your abacus.

OBV

Ten tens or 100 should be recorded on your abacus. When you used the rule clear 5, set 1 left to add 50, you cleared 5 tens and replaced it with ten tens. Clear your abacus.

---

19 This time let's work a problem in which you'll need to use the rule clear 5, set 1 left to add in the tens column. Set 64 on your abacus. To that number add 5 tens and five or 55. Read me your answer.

$$\begin{array}{r} 64 \\ +55 \\ \hline 119 \end{array}$$

Good. or The correct answer is 119.

Use your right index finger to clear your abacus.

Criterion Check

I'm going to read four problems which you should be able to work. When you've finished I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{r} 69 \\ +25 \\ \hline 94 \end{array} \qquad \begin{array}{r} 72 \\ +56 \\ \hline 128 \end{array} \qquad \begin{array}{r} 85 \\ +52 \\ \hline 137 \end{array} \qquad \begin{array}{r} 26 \\ +15 \\ \hline 41 \end{array}$$

---

20 This time let's find the sum of three, two-digit numbers. You'll need to use more than one rule of addition. Set 46 on your abacus. To that number add 35. Now, instead of clearing 81 add 14. Read me your answer.

$$\begin{array}{r} 46 \\ +35 \\ \hline 81 \\ +14 \\ \hline 95 \end{array}$$

Good. or Ninety-five is the correct answer.

Clear your abacus.

21

You've used the rule clear 5, set 1 left in the ones column and in the tens column. Listen as I read two statements. When I have finished set 1 or 2 on your abacus to show whether you think the first or second statement describes the use of that rule in the hundreds column.

1. You cannot use the rule clear 5, set 1 left in the hundreds column because in that case the column to the left stands for 1000. When the rule says clear 1 left it means clear 10.

*2. As you move to the left on an abacus each column is worth ten times as much as the one before it. Therefore, if you begin in the hundreds column to clear 5 and set 1 left you will have cleared 5 hundreds and set 10 hundreds or 1000.

The second statement describes the use of the rule in the hundreds column.

ANSWER STARRED

22

Here's a problem in which you can use the rule for adding 5 in the hundreds column. Set 734 on your abacus. To that number add 561.

The correct answer is 1,295. Did you get the correct answer?

Clear your abacus.

734
+561
1295

YES

23

Just for fun set 999 on your abacus. From that number subtract 175. Now, instead of clearing 824 add 555. Watch for a chance to use the rule clear 5, set 1 left. Look carefully above and below the bar when you read me your answer.

Good. or Your final answer should be 1,379.

Clear your abacus.

999
-175
824
+555
1379

Here's a problem in which there are four numbers to add. You'll need to use several rules of addition. Begin by setting 123 on your abacus. To that number add 654. Now add 52. Finally add 135.

$$\begin{array}{r} 123 \\ +654 \\ \hline 777 \\ +\ 52 \\ \hline 829 \\ +135 \\ \hline 964 \end{array}$$

24

The correct answer is 964. Did you get the correct answer?

Clear your abacus.

Criterion Check

I'm going to read seven problems which you should be able to work. When you've finished I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{rrrrrrr} 446 & 841 & 336 & 573 & 932 & 517 & 403 \\ +535 & +528 & +435 & +154 & +554 & +145 & +312 \\ \hline 981 & 1369 & 771 & 727 & 1486 & 662 & 715 \\ & & & & & & +145 \\ \cline{7-7} & & & & & & 860 \end{array}$$

LESSON FIFTEEN

It's time to begin another abacus lesson. Before we begin something new, I'm going to read each of the rules of abacus addition which you've learned to use. To show that you know what number can be added with each rule record that number on your
1 abacus. Listen carefully.

Clear 1, set 5	4
Clear 2, set 5	3
Clear 5, set 1 left	5
Clear 4, set 5	1
Clear 3, set 5	2

Fine. or Restatement of rule indicating its application.

Now, let's work a problem which will give you a chance to use all five of the rules you know. There is no number larger than 55 in the problem but it has nine different numbers in it. I will read the problem slowly so that you'll have time to work
2 accurately. Here's the problem.

1 + 4 + 5 + 14 + 1 + 34 - 55 + 2.

Your final answer should be 6. Did you get the correct answer? YES

Clear your abacus.

Set 4 on your abacus. You know that 4+6=10.
3 Make your abacus say 10. OBV

To show 10, you set one bead in the tens column. What did you do with the four
4 lower beads that were set at the bar? CLEARED THEM

Clear your abacus.

5 It is possible to add 6 by clearing 4 and setting 10. Here is a problem in which you can use that way of adding 6. Set 24 on your abacus. To that number add 1 ten and 6 or 16.

$$\begin{array}{r}24\\+16\\\hline 40\end{array}$$

Forty is the correct answer. Did you get the correct answer?

YES

Do not clear forty.

---

6 On an abacus each column to the left is worth ten times as much as the one before it. Therefore, instead of saying clear 4 and set 10 we can say clear 4 and set 1 left. Add 6 tens or 60 to the number that is recorded on your abacus. Ten tens or 100 should be recorded on your abacus.

$$\begin{array}{r}40\\+60\\\hline 100\end{array}$$

OBV

Clear your abacus.

---

7 Set the number 400 on your abacus. Now, add 6 hundreds by clearing 4 and setting one left. Read me your answer.

$$\begin{array}{r}400\\+600\\\hline 1000\end{array}$$

No comment. or The correct answer is 1000.

You may clear your abacus.

---

8 Set 4000 on your abacus. Add 6000 to that number and read me your answer.

The column in which you recorded your answer is called the 10,000s column. Clear 10,000 from your abacus.

$$\begin{array}{r}4000\\+6000\\\hline 10{,}000\end{array}$$

---

9 Set three 10,000's or 30,000 on your abacus. Now clear 30,000. Show me the finger you used to clear 30,000

RIGHT INDEX

Good. or You should have cleared 30,000 with your right index finger.

10 Set 20,000 on your abacus. Now, find the thousands column and set 4,000. Finally, add 341. Read me your answer. OBV

Use your right index finger to clear 24,341.

Criterion Check

I'm going to read eight numbers to you. I would like you to set those numbers on your abacus. Each of you will have different numbers to set.

GROUP A:	32,412	23,456	37,962	9,068
	12,238	48,296	10,986	2,697
GROUP B:	43,124	3,587	65,432	48,753
	7,098	14,347	26,385	15,760
GROUP C:	21,341	3,786	43,265	48,763
	8,079	14,146	37,387	13,875

This time I'm going to record eight numbers on your abacus. I would like you to read them to me. Each of you will have different numbers to read.

GROUP A	24,131	3,865	47,152	16,385
	8,706	11,246	48,753	12,797
GROUP B:	31,423	2,896	84,163	48,296
	6,709	13,125	37,962	12,570
GROUP C:	32,434	2,785	36,252	17,475
	7,605	12,127	49,825	14,687

---

11 In the first part of this lesson, you began to use the rule clear 4, set 1 left to add 6. In this problem you'll need to apply that rule twice. Set 44 on your abacus. To that number add 6 tens and 6 or 66.

$$\begin{array}{r} 44 \\ +66 \\ \hline 110 \end{array}$$

The correct answer is 110. Did you get the correct answer?

YES

Clear your abacus.

12	Set 3 on your abacus. Add 6 to 3. Did you need to use the rule clear 4, set 1 left to show that 3+6=9? No comment. or You should not have used the rule in that case.	NO
13	Instead of clearing 9, add 6 to it. Did you use the rule clear 4, set 1 left to add 6? No comment. or You should have used the rule in that case. Clear your abacus.	9 +6 15 YES
14	Set 29 on your abacus. To that number add 66. Look for a chance to use the rule, clear 4, set 1 left. Read me your answer. Good. or Ninety-five is the correct answer. You may clear your abacus.	29 +66 95
15	Try this problem. Set 59 on your abacus. Then, add 36. Read me your answer. Good. or The correct answer is 95. Clear your abacus so you'll be ready for another problem.	59 +36 95
16	This time let's add 3 numbers. Set 45 on your abacus. To 45 add 64. Instead of clearing 109 add 76. Read me your answer. I agree. or The correct answer is 185. Clear your abacus.	45 +64 109 + 76 185

Criterion Check

I'm going to read seven problems which you should be able to work independently. After you've finished I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

42	79	14	54	92	98	93
+62	+16	+56	+36	+61	+61	+61
104	95	70	90	153	159	154
			+65			+26
			155			180

---

17 This time let's work a problem which has three-digit numbers in it. You'll need to use more than one rule of addition. Set 291 on your abacus. To that number add 564. Look above and below the bar when you read your answer.

291
+564
855

Good. or The correct answer is 855.

Clear your abacus.

---

18 This problem has both addition and subtraction in it. Set 694 on your abacus. To that number add 161. Instead of clearing 855 subtract 150. Read me your final answer.

694
+161
855
-150
705

I agree. or Your final answer should be 705.

Clear your abacus.

---

19 Now, let's work a problem that has four-digit numbers in it. You'll need to use the ten thousands column to record your answer. Set 9,124 on your abacus. To that number add 6,636. Look carefully at your answer and read it to me.

9,124
+6,636
15,760

Good. or Your final answer should be 15,760.

Clear your abacus.

At the beginning of this lesson you worked a rather long problem which gave you an opportunity to use all you know about adding on the abacus. This problem is similar to that one. Here's the problem.

20 1 + 2 + 2 + 4 + 6 + 40 + 5 + 4 + 1

Your final answer should be 65. Did you add accurately?

YES

Clear your abacus.

Criterion Check

I'm going to read eight problems which you should be able to work. When you've finished I'll ask you to read your answer aloud. If you would like me to repeat a problem raise your hand.

$$\begin{array}{r} 44 \\ +35 \\ \hline 79 \\ +16 \\ \hline 95 \end{array} \quad \begin{array}{r} 45 \\ +61 \\ \hline 106 \\ +\ 85 \\ \hline 191 \end{array} \quad \begin{array}{r} 987 \\ -566 \\ \hline 421 \\ +654 \\ \hline 1075 \end{array} \quad \begin{array}{r} 5892 \\ +2064 \\ \hline 7956 \end{array} \quad \begin{array}{r} 5149 \\ +3246 \\ \hline 8395 \end{array} \quad \begin{array}{r} 5897 \\ +2065 \\ \hline 7962 \end{array}$$

$$\begin{array}{r} 444 \\ +615 \\ \hline 1059 \end{array} \quad \begin{array}{r} 1979 \\ -1016 \\ \hline 963 \\ +652 \\ \hline 1615 \end{array}$$

LESSON SIXTEEN*

1	You've had several abacus lessons. In those lessons you've learned to record numbers of up to five digits on the abacus. You've done some subtraction and learned to use six of the rules of addition.	NONE
2	Before we being something new, let's work a problem which will give you a chance to use what you've learned about the abacus. I'll read the problem which has 9 numbers in it, slowly, so that you'll have plenty of time to work accurately. 15 + 5 + 4 + 6 + 43 + 2 - 55 + 34 + 1 Your final answer should be 55. Was your answer correct? Clear your abacus.	YES
3	Now that you've worked the warm-up problem let's begin something new. In this lesson you'll learn a way of adding 7.	NONE
4	Set 3. With your index finger clear 3. Then set 10 with your thumb. What number did you add when you cleared 3 and set 10? Clear your abacus.	7
5	Let's work a simple problem in which you can add 7 by clearing 3 and setting 10. Set 13 on your abacus. Then, add 7 to it. Twenty is the correct answer. Is that number recorded on your abacus? Clear your abacus.	$\begin{array}{r}13\\+\ 7\\\hline 20\end{array}$ YES

You've learned that on an abacus each column to the left is worth ten times as much as the one before it. Therefore, instead of saying clear 3, set 10, we can say clear 3, set 1 left.

I've given you a sheet with three problems on it. The correct answers have been given. Work each problem for yourself. Then circle the one problem in which you used the idea that a column to the left is worth ten times as much as the one before it.

6 ANSWER STARRED

42 + 57 = 99
*300 + 700 = 1000
213 + 75 = 288

You should have marked the second problem.

Set 1 on your abacus. Now add 7 to it.
Did you use the rule clear 3, set 1 left
7 to show that 1 + 7 = 8? NO

Do not clear 8 from your abacus.

Try to use the rule clear 3, set 1 left to see if it will help you show that 8 + 7 = 15. Did the rule work?

8

No comment. or The rule should have worked.

YES

Clear your abacus.

Let's work a problem in which you can use the rule clear 3, set 1 left twice. Set 38 on your abacus. Add 77 to it. Read
9 me your answer.

$$\begin{array}{r}38\\+77\\\hline 115\end{array}$$

Fine. or The correct answer is 115.

Clear your abacus.

Set the number 4 on your abacus. Now add 7 to it. Did you use the rule clear 3, set 1 left to show that 4 + 7 = 11?

10 No comment. or You should have used the rule in that case.

YES

Clear your abacus.

Let's use your abacus to show that 9 + 7 = 16. Set 9 and then add 7 to it. Did the rule work in that case?

11 No comment. or It should have worked.

YES

Clear your abacus.

Here's a problem that will give you two chances to use the rule for adding 7. First set 84. Then add 77 to it. Read

12 me your answer.

$$\begin{array}{r} 84 \\ +77 \\ \hline 161 \end{array}$$

Good. or The correct answer is 161.

Clear your abacus.

Criterion Check

I'm going to read five problems with you should be able to work. I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{rrrrr} 23 & 46 & 18 & 98 & 35 \\ +17 & +72 & +17 & +71 & +74 \\ \hline 40 & 118 & 35 & 169 & 109 \end{array}$$

13 I have given you a sheet which has three problems on it. The correct answers have been given. Work each problem for yourself. Then circle the problem or problems in which you used the rule clear 3, set 1 left to add 7.

ANSWERS STARRED

*34 + 57 = 91
*86 + 73 = 159
25 + 72 = 97

You should have circled the first two problems.

---

14 I'm going to read three rules which explain how to add on the abacus. To show that you know what number can be added with each rule set that number on your abacus.

Clear 5, set 1 left	5
Clear 4, set 1 left	6
Clear 3, set 1 left	7

No comment. or Restatement of rule in sentence indicating its application.

---

15 Here's a problem in which you'll need to use more than one rule of addition. Set 99 on your abacus. To that number add 67. Read me your answer.

$$\begin{array}{r} 99 \\ +67 \\ \hline 166 \end{array}$$

Fine. or Your answer should be 166.

Remember to use your right index finger when you clear your abacus.

---

16 Let's add several numbers this time. Begin by setting the number 73. To that number add 17. Instead of clearing 90 add 78. Finally, add 51. Read me your answer.

$$\begin{array}{r} 73 \\ +17 \\ \hline 90 \\ +78 \\ \hline 168 \\ +\ 51 \\ \hline 219 \end{array}$$

I agree. or Your answer should be 219.

Clear your abacus.

Criterion Check

I'm going to read five problems which you should be able to work. After you've finished I'll ask you to read your answer aloud. If you would like me to repeat a problem raise your hand.

$$\begin{array}{r} 64\\ +57\\ \hline 121 \end{array} \qquad \begin{array}{r} 34\\ +76\\ \hline 110 \end{array} \qquad \begin{array}{r} 94\\ +73\\ \hline 167 \end{array} \qquad \begin{array}{r} 29\\ +17\\ \hline 46\\ +62\\ \hline 108 \end{array} \qquad \begin{array}{r} 58\\ +25\\ \hline 83\\ +\ 7\\ \hline 90 \end{array}$$

---

17 This time let's work a problem that has three-digit numbers in it. Watch for a chance to use more than one rule of addition. Set 148 on your abacus. To that number add 2 hundreds, 7 tens and 5 or 275. Read me your answer.

$$\begin{array}{r} 148\\ +275\\ \hline 423 \end{array}$$

Good. or The correct answer is 423.

Clear your abacus.

---

18 This problem has both addition and subtraction in it. Set 249 on your abacus. To that number add 567. The number 816 should be recorded on your abacus. Now, subtract 305.

$$\begin{array}{r} 249\\ +567\\ \hline 816\\ -305\\ \hline 511 \end{array}$$

Your final answer should be 511. Did you get the correct answer?

YES

Clear your abacus.

---

19 Now let's try a problem in which there are large numbers. You'll need to use the ten thousands column to record your answer. Set 9,214 on your abacus. To that number add 7,531.

$$\begin{array}{r} 9{,}214\\ +7{,}531\\ \hline 16{,}745 \end{array}$$

The correct answer is 16,745. Did you get the correct answer?

YES

Clear your abacus.

Criterion Check

I'm going to read five problems which you should be able to work. After you've finished I'll ask you to read your answer aloud. If you would like me to repeat a problem raise your hand.

$$\begin{array}{r} 159 \\ +437 \\ \hline 596 \end{array} \quad \begin{array}{r} 299 \\ +576 \\ \hline 875 \end{array} \quad \begin{array}{r} 754 \\ +537 \\ \hline 1291 \end{array} \quad \begin{array}{r} 244 \\ +427 \\ \hline 671 \\ +253 \\ \hline 924 \end{array} \quad 1 + 4 + 4 + 6 + 5 + 3 + 7 = 30$$

LESSON SEVENTEEN

It's time for another abacus lesson. As usual, let's begin by working a problem in which you can use what you've learned about adding with an abacus. Get ready to add: 4 + 7 + 4 + 5 + 34 + 6 - 50 + 34 + 20 + 1.

1

Your final answer should be 65. Did you get the correct answer?

Clear your abacus so you'll be ready for something new. YES

2 Set 2 on your abacus. Now clear two and set 10. What number did you add when you cleared 2 and set 10? 8

No comment. or You added 8.

3 Let's work a problem in which you must add 8 by clearing 2 and setting 10. Set 22 on your abacus. To that number add 1 ten and 8 or 18. Read me your answer.

$$\begin{array}{r} 22 \\ +18 \\ \hline 40 \end{array}$$

Good. or Forty is the correct answer.

Clear your abacus.

4 Set 3 on your abacus. Will the rule clear 2, set 10 help you show that 3 + 8 = 11? Try it before you answer.

I agree. or Yes, it will work.

YES

Clear your abacus.

5 Because each abacus column to the left is worth ten times as much as the one before it we can use the phrase clear 2, set 1 left to explain what you did to add 8. Set 25 on your abacus. To that number add 84. Use the idea that 10 tens equals 100 to add 8. The correct answer is 109. Was your answer correct?

$$\begin{array}{r} 25 \\ +84 \\ \hline 109 \end{array}$$

YES

Clear your abacus.

6

Set 200 on your abacus. Now add 800 to it. Did you use the idea that 10 tens equal 100 or that 10 100's equal 1000 when you cleared 2 and set 1 left?

I agree. or You used the idea that ten 100's equal 1000.

Clear your abacus.

ten 100's equal 1000

7

Here's a problem in which you'll get two chances to use the rule clear 2, set 1 left to add 8. Set 74 on your abacus. To that number add 88. Read me your answer.

Fine. or Your answer should be 162.

Clear your abacus.

$$\begin{array}{r} 74 \\ +88 \\ \hline 162 \end{array}$$

8

Set 33 on your abacus. To that number add 85. Look above and below the bar when you read me your answer.

Good. or Your answer should be 118.

Clear your abacus.

$$\begin{array}{r} 33 \\ +85 \\ \hline 118 \end{array}$$

Criterion Check

I'm going to read four problems which you should be able to work independently. When you've finished I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{r} 12 \\ +68 \\ \hline 80 \end{array} \qquad \begin{array}{r} 35 \\ +82 \\ \hline 117 \end{array} \qquad \begin{array}{r} 59 \\ +28 \\ \hline 87 \end{array} \qquad \begin{array}{r} 57 \\ +38 \\ \hline 95 \end{array}$$

I have given you a sheet that has three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used the rule clear 2, set 1 left to add 8.

9 *52 + 28 = 80
*42 + 86 = 128
12 + 86 = 98

ANSWERS STARRED

You should have marked the first two problems.

This time let's add several numbers in a series. You'll need to use more than one rule of addition. Begin by setting 42 on your abacus. To that number add 38.
10 Instead of clearing 80, add 88. Read me your answer.

$$\begin{array}{r}42\\+38\\\hline 80\\+88\\\hline 168\end{array}$$

Good. or Your final answer should be 168.

Clear your abacus.

Here's a problem similar to the one you worked at the beginning of this lesson. You'll need to use several of the rules of addition which you've learned.
12 + 4 +5 + 38 + 8

11

Your final answer should be 67. Did you get the correct answer?

YES

Clear your abacus.

There are three-digit numbers in this problem. Set 248 on your abacus. To that number add 1 hundred, 7 tens and
12 8 or 178.

$$\begin{array}{r}248\\+178\\\hline 426\end{array}$$

426 is the correct answer. Did you get the correct answer?

YES

Clear 426.

Criterion Check

I'm going to read six problems which you should be able to work. When you've finished, I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

23	34	839	767	94	53
+87	+41	+866	+518	+68	+43
110	75	1695	1285	162	96
	+82				+85
	157				181

---

13 This problem has both addition and subtraction in it. Begin by setting 968 on your abacus. From that number subtract 615. Now add 328.

$$\begin{array}{r} 968 \\ -615 \\ \hline 353 \\ +328 \\ \hline 681 \end{array}$$

Your final answer should be 681. Was your answer correct?

YES

Clear your abacus.

---

14 Here's a problem which has both addition and subtraction in it. Set 2,743 on your abacus. To that number add 4,185. Now subtract 5,622.

$$\begin{array}{r} 2743 \\ +4185 \\ \hline 6928 \\ -5622 \\ \hline 1306 \end{array}$$

Your final answer should be 1,306. Did you get the correct answer?

Clear your abacus.

Criterion Check

I'm going to read five problems which you should be able to work. When you've finished I'll ask one of you to read your answer. Raise your hand if you'd like me to repeat a problem.

219	8,432	5719	193	4,192
+415	+5,181	+2827	+574	+8,765
634	13,613	8546	767	12,957
+128		-6525	+518	
762		2021	1285	

LESSON EIGHTEEN*

1	You've learned special rules for adding 1, 2, 3, 4, 5, 6, 7 and 8 on the abacus. It should be easy to guess that in this lesson you'll learn a rule for adding 9.	NONE
2	Before learning the new rule, here's a problem that will give you a chance to use almost all the rules you've learned so far. Get ready to add. 9 + 8 + 5 + 4 - 25 + 3 + 7 - 10 + 3 + 6 + 3 + 3. Your final answer should be 16. Did you get the correct answer? Clear your abacus.	YES
3	A rule for adding 6 is clear 4, set 1 left. A rule for adding 7 is clear 3, set 1 left. A rule for adding 8 is clear 2, set 1 left. What is a rule for adding 9? Good. or To clear 1 and set 1 left is a way of adding 9.	CLEAR 1, SET 1 LEFT
4	Here's a problem in which you can use the rule clear 1, set 1 left to add 9. Set 71 on your abacus. To that number add 19. Do not clear 90 from your abacus.	71 +19 90 OBV
5	Add 9 tens to the 90 which is recorded on your abacus. The number 180 should appear on your abacus. Did you add correctly? Clear your abacus.	90 +90 180 YES

Instead of saying that the rule for adding 9 is clear 1, set 10, we say it is clear 1 and set 1 left. I'm going to read two sentences. When I have finished set a 1 or 2 on your abacus to show whether you think the first or second sentence best explains why we say clear 1 left.

1. It's easier to say clear 1, set 1 left than to say clear 1, set 10.

6 ANSWER STARRED

*2. As you move to the left each column on an abacus is worth 10 times as much as the one before it. To say clear 1, set 1 left is a way of saying that a rule will work in the ones column, the tens column, the hundreds column and the thousands column.

The second statement I read is true.

Here's a problem in which you'll need to use the rule clear 1, set 1 left in two different columns. Set 28 on your abacus. To that number add 99. Read me your answer.

$$\begin{array}{r} 28\\ +99\\ \hline 127\end{array}$$

7 I agree. or I disagree your answer should be 127.

Use your right index finger to clear your abacus.

Criterion Check

I'm going to read five problems which you should be able to work. When you've finished, I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{rrrrr} 51 & 88 & 73 & 61 & 93\\ +39 & +91 & +19 & +95 & +95\\ \hline 90 & 179 & 92 & 156 & 188\end{array}$$

8 In this problem you'll need to use the rule for adding 9 and one other rule of abacus addition. Set 43 on your abacus. To that number add 89. Read me your answer.

43
+89
132

Good. or The correct answer is 132.

Clear your abacus.

---

9 This time let's add three numbers. Begin by setting 24. To that number add 39. Then add 27.

Your final answer should be 90. Did you get the correct answer?

24
+39
63
+27
90

Clear your abacus.

Yes

---

10 I'm going to read four rules of addition. To show that you know what number can be added with each rule set that number on your abacus.

Clear 5, set 1 left	5
Clear 2, set 5	3
Clear 1, set 5	4
Clear 3, set 5	2

Good. or Restatement of rule indicating its application.

---

11 Here's a problem that has several numbers in it. I'll read slowly so that you'll have plenty of time to work accurately.
4 + 9 + 4 + 5 + 2 + 7 + 83 + 31 + 60 + 4
+ 6

Your final answer should be 215. Was your answer correct?

Clear your abacus.

YES

Criterion Check

I'm going to read five problems which you should be able to work. When you've finished, I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{rrrrr} 14 & 83 & 49 & 91 & 47 \\ +43 & +92 & +19 & +63 & +78 \\ \hline 57 & 175 & 68 & 154 & 125 \\ +59 & +51 & +98 & +51 & +94 \\ \hline 116 & 226 & 166 & 205 & 219 \end{array}$$

12

There are three-digit numbers in this problem. Set 664 on your abacus. To that number add 259. Read me your answer.

Good. or The correct answer is 923.

Clear your abacus so you'll be ready for another problem.

$$\begin{array}{r} 664 \\ +259 \\ \hline 923 \end{array}$$

13

Try this problem. Set 298 on your abacus. To that number add 169. Now add 652.

The final answer should be 1,119. Was your answer correct?

Clear your abacus.

$$\begin{array}{r} 298 \\ +169 \\ \hline 467 \\ +652 \\ \hline 1119 \end{array}$$

YES

14

This problem has both addition and subtraction in it. Set 799 on your abacus. Now subtract 113. Now add 179. Look above and below the bar when you read your answer.

Good. or Your final answer should be 865.

Clear 865.

$$\begin{array}{r} 799 \\ -113 \\ \hline 686 \\ +179 \\ \hline 865 \end{array}$$

Get ready for a problem in which there are four-digit numbers. Set 9,643 on your abacus. To that number add 9,212. Read me your answer.

15

Good. or Your final answer should be 18,855.

$$\begin{array}{r} 9{,}643 \\ +9{,}212 \\ \hline 18{,}855 \end{array}$$

You may clear your abacus.

Criterion Check

I'm going to read five problems which you should be able to work. When you've finished, I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

4,991	392	329	1942	8,614
+6,791	+579	+298	- 620	+9,246
11,782	971	627	1322	17,860
			+ 738	
			2060	

LESSON NINETEEN

1 You've learned a rule for adding all of the numbers from 1 to 9 on the abacus. Here's a problem in which you can use all nine of those rules. 4 + 2 + 9 + 5 + 34 + 1 - 55 + 8 + 8 -15 + 4 + 3 + 7 + 4 + 6.

Your final answer should be 25. Did you get the correct answer?

YES

Clear your abacus.

Set 9 on your abacus. You know that 9 + 1 = 10. Make your abacus say 10.

2 To record 10 you cleared 9 and set a ten-bead to the bar. We can say that you added 1 by clearing 9 and setting 1 left. OBV

To add 1 you must clear 9 before you set a bead to the left. Show me the finger you should use to clear 9.

3 Good. or You should use your right index finger. RIGHT INDEX

Clear your abacus so you'll be ready for a new problem.

Set 29 on your abacus. To that number add 5 tens and 1 or 51. Use the rule clear 9, set 1 left when you add in the one's column.

4

$$\begin{array}{r}29\\+51\\\hline 80\end{array}$$

The number 80 should be recorded on your abacus.

Clear your abacus.

5 Set 92 on your abacus. To that number add 1 ten and 6 or 16. You'll need to use the rule clear 9, set 1 left to add in the tens column.

$$\begin{array}{r}92\\+16\\\hline 108\end{array}$$

The correct answer is 108. Did you get the correct answer?

YES

Clear your abacus.

---

6 It is possible to show 9 in any column on the abacus. To set a bead to the left is a way of counting 10. As you work this problem think about whether you are using that idea to add in the ones column, the tens column, the hundreds column, or the thousands column. Set 945 on your abacus and then add 153 to it.

$$\begin{array}{r}945\\+153\\\hline 1098\end{array}$$

Your answer should be 1,098. Did you use the rule for adding 1 in the ones, tens, hundreds, or thousands column?

Good. or You should have used it in the hundreds column.

HUNDREDS

Clear your abacus.

Criterion Check

I'm going to read five problems which you should be able to work. When you have finished, I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{r}29\\+51\\\hline 80\end{array}\qquad\begin{array}{r}96\\+13\\\hline 109\end{array}\qquad\begin{array}{r}19\\+61\\\hline 80\end{array}\qquad\begin{array}{r}97\\+11\\\hline 108\end{array}\qquad\begin{array}{r}59\\+31\\\hline 90\end{array}$$

---

7 You've learned two rules for adding one. One rule says clear 4, set 5. What is the other rule for adding 1?

CLEAR 9, SET 1 LEFT

Good. of To clear 9 and set 1 left is a way of adding 1.

8 To work this problem you'll need both of the rules for adding one. Set 49 on your abacus. To that number add 1 ten and 1 or 11. Read me your answer.

Good. or Sixty is the correct answer.

Clear your abacus.

49
+11
60

9 It's important that you be able to decide what rule you should use. I've given you a sheet that has three problems on it. The correct answer have been given. Work each problem for yourself and then circle the 1 problem or problems in which you used the rule clear 9, set 1 left to add 1.

46 + 12 = 58
*29 + 51 = 80
26 + 71 = 97

You should have marked the second problem.

ANSWER STARRED

10 This time let's add several numbers. You'll need to use more than one rule of addition. Set 54 on your abacus. To that number add 41. Now add 15.

Your final answer should be 110. Did you get the correct answer?

Clear your abacus.

54
+41
95
+15
110

YES

11 Here's another problem in which there are several numbers. Set 69 on your abacus. To that number add 21. Now, add 64. Then add 33. Read me your answer.

I agree. or Your final answer should be 187.

You may clear your abacus.

69
+21
90
+64
154
+33
187

Criterion Check

I'm going to read five problems which you should be able to work. When you've finished I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

34	89	46	49	39
+58	+71	+73	+46	+91
92	160	119	95	130
+13	+ 91	+ 41	+15	+ 82
105	251	160	110	212

---

12 There are three-digit numbers in this problem. Set 909 on your abacus. To that number add 127. Look carefully at your abacus before you read your answer.

909
+127
1036

Good. or Your answer should be 1,036.

Clear your abacus.

---

13 To work this problem you'll need to use more than one rule of addition. Set 289 on your abacus. To that number add 181. Look above and below the bar when you read your answer.

289
+181
470

Good. or The correct answer is 470.

Clear your abacus.

---

14 Try this problem. Set 496 on your abacus. To that number add 313. Read me your answer.

496
+313
809

Good. or The correct answer is 809.

Clear your abacus.

15	This problem has both addition and subtraction in it. Set 499 on your abacus. To that number add 171. Now subtract 150. Your final answer should be 520. Was your answer correct? Clear your abacus.	499 +171 670 -150 520 YES
16	This problem has four-digit numbers in it. Set 9,454 on your abacus. To that number add 1,241. Look carefully at your abacus before you read me your answer. Good. or The correct answer is 10,695. Clear your abacus.	9,454 +1,241 10,695
17	The numbers in this problem have three and four digits. You'll need to think carefully as you work. Begin by setting 5,392 on your abacus. To that number add 2,816. Now add 635. Your final answer should be 8,843. Did you get the correct answer? Clear your abacus.	5392 +2816 8208 + 635 8843 YES

Criterion Check

I'm going to read five problems which you should be able to work independently. I'll ask you to read your results aloud. Raise your hand if you would like me to repeat a problem.

498	993	9,687	38	874
+318	+712	+1,285	+89	+816
816	1705	10,972	127	1690
			649	
			776	

LESSON TWENTY

1	This lesson is a little different from most of the lessons you've had. Instead of learning a new rule you'll use two of the rules you've already learned in a new way.	NONE
2	I've given you a sheet with three phrases on it which explain how to add on the abacus. Read each phrase and then circle every phrase which explains a way of adding 1. *Clear 4, set 5 Clear 2, set 5 *Clear 9, set 1 left To clear 4, set 5 and to clear 9, set 1 left are two ways of adding 1.	ANSWERS STARRED
3	Set 49 on your abacus. If I asked you to add 1 in the ones column what rule would you use to help you? Good. or You could add 1 by clearing 9 and setting 1 left.	CLEAR 9, SET 1 LEFT
4	Clear 9 in the one's column. Is it possible to set 1 left by setting a ten-bead to the bar?	NO
5	Because there is not a ten-bead you can set to the bar, you will need to use a special rule to help you add 1 in the tens column. Do it. The number 50 should be recorded on your abacus. What rule did you use to add 1 in the ten's column? Good. or You should have used the rule clear 4, set 5 to help you add 1.	CLEAR 4, SET 5

6	Some of the rules of addition say set 1 left. For some problems you'll need to use a rule for adding 1 to help you set 1 left. This lesson should help you learn to do that.	NONE
7	Set 39 on your abacus. To that number add 1 ten and 6 or 16. You'll need to use the rule clear 4, set 1 left to add 6. When you're ready to set 1 left use the rule clear 4, set 5. The correct answer is 55. Was your answer correct? Clear your abacus.	$\begin{array}{r} 39 \\ +16 \\ \hline 55 \end{array}$ YES
8	Set 45 on your abacus. Now, add 5. Think carefully about what you must do when it is time to set 1 left. Clear 50 from your abacus.	$\begin{array}{r} 45 \\ +\ 5 \\ \hline 50 \end{array}$
9	I have given you a sheet which has three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used the rule clear 4, set 5 when it was time to set 1 left. 34 + 57 = 91 27 + 65 = 92 *13 + 38 = 51 You should have circled the third problem.	ANSWER STARRED

Criterion Check

I'm going to read five problems which you should be able to work. I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

29	38	18	44	34
+26	+18	+37	+ 6	+19
55	56	55	50	53

10	A rule for adding 1 is clear 9, set 1 left. Look for a chance to use that rule in this problem. 64 + 24 + 1 + 1 Your final answer should be 90. Was your answer correct? Clear your abacus.	64 +24 88 + 1 89 + 1 90 YES
11	In the first part of this lesson you used the rule clear 4, set 5 to help you set 1 left. It is also possible to use the rule clear 9, set 1 left to help you set 1 left. Be sure your abacus is clear so that you can work a problem which uses that idea.	NONE
12	Set 99 on your abacus. Add 7 to it. You'll need to use the rule clear 3, set 1 left to add 7. When you're ready to set 1 left, think of the rule clear 9, set 1 left. The correct answer is 106. Was your answer correct? Clear your abacus.	99 + 7 106 YES
13	Here's another problem in which you'll need to use a rule for adding 1 when it's time to set 1 left. Set 78 on your abacus. To what number add 2 tens and 9 or 29. Read me your answer. Good. or Your final answer should be 107. Clear your abacus.	78 +29 107

Criterion Check

I'm going to read five problems which you should be able to work. When you've finished I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

64	189	98	69	58
+38	+ 19	+ 7	+36	+48
102	208	105	105	106

---

14 I have given you a sheet that has three problems on it. The correct answers have been given. Use your abacus to work each problem for yourself and then circle the problem or problems in which you used a rule for adding 1 when it was time to set 1 left. ANSWERS STARRED

*94 + 8 = 102
45 + 62 = 107
*22 + 29 = 51

You should have marked the first and third problems.

---

15 This problem will give you two chances to use the rules for adding 1 in the new way. The problem is a long one, but I'll read it slowly so that you'll have plenty of time to work accurately. 44 + 8 + 3 + 5 + 34 + 7 + 4 + 4 + 1.

Your final answer should be 110. Was your answer correct?

Clear your abacus. YES

---

16 There are three-digit numbers in this problem. You'll need to use a rule for adding one when it is time to set 1 left. Set 195 on your abacus. To that number add 313. Read me your answer.

$$\begin{array}{r} 195 \\ +313 \\ \hline 508 \end{array}$$

I agree. or Your answer should be 508.

Begin in the hundreds column when you clear your abacus.

---

17 Try this problem which has both addition and subtraction in it. Set 437. To that number add 365. Now subtract 202. Read me your answer.

$$\begin{array}{r} 437 \\ +365 \\ \hline 802 \\ -202 \\ \hline 600 \end{array}$$

Good. or Six hundred is the correct answer.

Clear your abacus.

---

18 This time you'll need to use a rule for adding 1 to set 1 left in a problem in which there are four-digit numbers. Set 4,885 on your abacus. To that number add 2,172. Read me your answer.

$$\begin{array}{r} 4885 \\ +2172 \\ \hline 7057 \end{array}$$

Good. or The correct answer is 7,057.

Clear your abacus.

Criterion Check

I'm going to read five problems which you should be able to work. When you've finished I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{rrrrr} 1831 & 348 & 592 & 393 & 7{,}772 \\ +3822 & +958 & +452 & +163 & +2{,}813 \\ \hline 5653 & 1306 & 1044 & 556 & 10{,}585 \end{array}$$

8 Here's a problem in which you can use the rule clear 8, set 1 left to add 2. Set 28 on your abacus. To that number add 5 tens and 2 or 52.

$$\begin{array}{r} 28 \\ +52 \\ \hline 80 \end{array}$$

Do not clear 80.

9 Add 2 tens or 20 to the 80 which is set on your abacus. Your abacus should say 100.

OBV

Clear your abacus.

10 Try this problem. Set 18 on your abacus. To that number add 7 tens and 2 or 72. Look above and below the bar when you read me your answer.

$$\begin{array}{r} 18 \\ +72 \\ \hline 90 \end{array}$$

Good. or Ninety is the correct answer.

Clear your abacus.

11 Look for a chance to use the rule clear 8, set 1 left as you work this problem. Set 187 on your abacus. To that number add 222.

$$\begin{array}{r} 187 \\ +222 \\ \hline 409 \end{array}$$

The correct answer is 409. Did you get the correct answer.

YES

Clear your abacus.

12 You've learned that clearing 8 and setting 1 left is a way of adding 2. Set 9 on your abacus. Then, try that rule to see if it will help you show that 9 + 2 = 11. Did the rule work?

YES

Clear your abacus

13 Here's another problem in which you can practice the rule clear 8, set 1 left. Set 69 on your abacus. To that number add 2 tens and 2 or 22. Read me your answer.

$$\begin{array}{r} 69 \\ +22 \\ \hline 91 \end{array}$$

Good. or The correct answer is 91.

Clear your abacus.

Criterion Check

I'm going to read five problems which you should be able to work. When you've finished I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{r} 85 \\ +23 \\ \hline 108 \end{array} \qquad \begin{array}{r} 59 \\ +32 \\ \hline 91 \end{array} \qquad \begin{array}{r} 98 \\ +22 \\ \hline 120 \end{array} \qquad \begin{array}{r} 78 \\ +12 \\ \hline 90 \end{array} \qquad \begin{array}{r} 95 \\ +23 \\ \hline 118 \end{array}$$

---

14 You've learned two rules for adding two. One rule says clear 3, set 5 and the other says clear 8 set 1 left. You'll need to use both of them to work this problem. Set 496 on your abacus. To that number add 223.

$$\begin{array}{r} 496 \\ +223 \\ \hline 719 \end{array}$$

Your final answer should be 719. Was your answer correct?

Clear your abacus. YES

---

15 I have given you a sheet with three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used a special rule for adding two.

ANSWERS STARRED

*38 + 52 = 90
46 + 52 = 98
*84 + 25 = 109

You should have circled the first and third problems.

16 This time let's work a problem in which there are several numbers. You'll need to use more than one rule of addition. Set 49 on your abacus. To that number add 12. Now add 53.

$$\begin{array}{r} 49 \\ +12 \\ \hline 61 \\ +53 \\ \hline 114 \end{array}$$

Your final answer should be 114. Was your answer correct?

Clear your abacus.

YES

---

17 Think carefully as you work this problem. Set 83 on your abacus. To that number add 21. Instead of clearing your abacus add 48. Look above and below the bar when you read your answer.

$$\begin{array}{r} 83 \\ +21 \\ \hline 104 \\ +48 \\ \hline 152 \end{array}$$

Good. or The correct answer is 152.

Clear your abacus.

Criterion Check

I'm going to read five problems and will ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{rrrrr} 408 & 47 & 92 & 99 & 29 \\ +272 & +21 & +67 & +72 & +22 \\ \cline{1-5} 680 & 68 & 159 & 171 & 51 \\ & +32 & +\ 51 & +\ 94 & +29 \\ \cline{2-5} & 100 & 210 & 265 & 80 \end{array}$$

---

18 There are three-digit numbers in this problem. Set 309 on your abacus. To that number add 542. Look above and below the bar when you read your answer.

$$\begin{array}{r} 309 \\ +542 \\ \hline 851 \end{array}$$

Good. or The correct answer is 851.

Clear your abacus.

19	Try this problem. Set 941 on your abacus. To that number add 216. Then add 258. Your final answer should be 1,415. Was your answer correct? Clear your abacus.	941 +216 1157 +258 1415 YES
20	This problem has four-digit numbers in it. You'll need to use several rules of addition. Set 1,982 on your abacus. To that number add 4,126. Read me your answer. Good. or Your final answer should be 6,108. Clear your abacus.	1982 +4126 6108
21	Let's work a problem that has both addition and subtraction in it. Begin by setting 3,989 on your abacus. To that number add 3,256. Now subtract 6,130. Your final answer should be 1,115. Was your answer correct? Clear your abacus.	3989 +3256 7245 -6130 1115 YES

Criterion Check

I'm going to read five problems and will ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

5489 +1927 7416	8,889 +2,791 11,680	423 +256 679 +582 1261	2829 +5242 8071	350 +331 681 +924 1605

LESSON TWENTY-TWO

Before learning the new rule which we'll practice in this lesson, let's try a problem which will give you a chance to use most of the rules you've already learned. Here's the problem. 7 + 5 + 3 + 44 + 6 + 4 + 1 +
1 80 - 150 + 9 + 7 + 92 + 2 - 100 + 34 + 12.

Your final answer should be 56. Did you get the correct answer?

Clear your abacus.

Set 7 on your abacus. Now clear 7 and set 1 left so that the number 10 will appear on your abacus. What number did you add when
2 you cleared 7 and set 1 left? 3

No comment. or You added 3.

Clear your abacus.

Here's a problem in which you will need to use the rule clear 7, set 1 left to add 3. Set 57 on your abacus. To that number add
3 23. Read me your answer.

$$\begin{array}{r} 57 \\ +23 \\ \hline 80 \end{array}$$

Good. or Eighty is the correct answer.

Clear your abacus.

Try this problem and watch for a chance to use the rule clear 7, set 1 left as a way of adding 3. Set 75. To that number add 34. Read me your answer.

$$\begin{array}{r} 75 \\ +34 \\ \hline 109 \end{array}$$

4

Good. or The correct answer is 109.

Clear your abacus.

I'm going to read three rules of addition. Raise your hand each time you hear a rule which explains a way of adding 3.

5 *Clear 3, set 1 left
*Clear 2, set 5 ANSWERS
Clear 3, set 5 STARRED

To clear 7, set 1 left and to clear 2 set 5 are two ways of adding 3.

Here's a problem in which you'll need to use both of the rules for adding 3. Set 73 on your abacus. To that number add 31. The number 104 should appear on your
6 abacus. Add 63 to it.

$$\begin{array}{r}73\\ +31\\ \hline 104\\ +\ 63\\ \hline 167\end{array}$$

Your final answer should be 167. Did you get the correct answer?

YES

Clear your abacus.

Set 8 on your abacus. Can you use the rule clear 7, set 1 left to show that 8 + 3 = 11? Try it before you answer.

7

No comment. or The rule should work.

YES

Clear your abacus.

Try this problem. Set 68 on your abacus. To that number add 2 tens and 3 or 23. Read me your answer.

8

$$\begin{array}{r}68\\ +23\\ \hline 91\end{array}$$

Good. or The correct answer is 91.

Clear your abacus.

This time begin by setting the number 9 on your abacus. The rule you've been practicing should help you show that 9 + 3 = 12.
9 Try it. OBV

Clear your abacus so you'll be ready for another problem.

---

10 In this problem you'll need to use the rule clear 7, set 1 left more than one time. Set 198 on your abacus. To that number add 533.

$$\begin{array}{r} 198 \\ +533 \\ \hline 731 \end{array}$$

The correct answer is 731. Was your answer correct?

Clear your abacus. YES

---

11 You've just used the rule clear 7, set 1 left to add 3. What is the other rule for adding 3?

CLEAR 2, SET 5

Good. or To clear 2 and set 5 is a way of adding 3.

---

12 Here's a problem in which you'll need to use both rules for adding 3. Set 96 on your abacus. To that number add 31. Now add 33.

$$\begin{array}{r} 96 \\ +31 \\ \hline 127 \\ +\ 33 \\ \hline 160 \end{array}$$

Your final answer should be 160. Did you get the correct answer?

Clear your abacus YES

Criterion Check

I'm going to read five problems. I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{r} 67 \\ +13 \\ \hline 80 \end{array} \qquad \begin{array}{r} 858 \\ +243 \\ \hline 1101 \end{array} \qquad \begin{array}{r} 19 \\ +13 \\ \hline 32 \\ +53 \\ \hline 85 \end{array} \qquad \begin{array}{r} 97 \\ +31 \\ \hline 128 \end{array} \qquad \begin{array}{r} 199 \\ +233 \\ \hline 432 \end{array}$$

---

13 To work this problem you'll need a rule for adding 1 to help you set 1 left. Set 438 on your abacus. To that number add 363. Be sure you look above and below the bar when you read your answer.

$$\begin{array}{r} 438 \\ +363 \\ \hline 801 \end{array}$$

Good. or The correct answer is 801.

Clear your abacus.

Try this problem. Begin by setting 593 on your abacus. Then add 333.

14 Your final answer should be 926. Did you get the correct answer?

$$\begin{array}{r} 593 \\ +333 \\ \hline 926 \end{array}$$

Clear your abacus.

YES

I've given you a sheet which has three problems on it. The correct answer have been given. Work each problem for youself and then circle the problem or problems in which you used a special rule for adding 3.

15

*804 + 313 = 1117
*693 + 233 = 926
651 + 333 = 984

ANSWERS STARRED

You should have marked the first two problems.

In this problem you'll need to use several rules of addition so work carefully. Begin by setting 94 on your abacus. To that number add 13. Now add 43.

16 Your final answer should be 150. Was your answer correct?

$$\begin{array}{r} 94 \\ +13 \\ \hline 107 \\ +\ 43 \\ \hline 150 \end{array}$$

YES

Clear your abacus.

Criterion Check

I'm going to read five problems which you should be able to work. When you've finished I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{r} 879 \\ -651 \\ \hline 228 \\ +353 \\ \hline 581 \end{array} \qquad \begin{array}{r} 169 \\ +233 \\ \hline 402 \end{array} \qquad \begin{array}{r} 28 \\ +23 \\ \hline 51 \\ +59 \\ \hline 110 \end{array} \qquad \begin{array}{r} 571 \\ +435 \\ \hline 1006 \end{array} \qquad \begin{array}{r} 99 \\ +36 \\ \hline 135 \\ +\ 74 \\ \hline 209 \end{array}$$

I'm going to read four rules of abacus addition. To show that you know what number can be added with each rule set that number on your abacus.

	Clear 1, set 1 left	9
17	Clear 2, set 1 left	8
	Clear 3, set 1 left	7
	Clear 9, set 1 left	1

Good. or Restatement of rule indicating its application.

This problem has three-digit numbers in it. Set 548 on your abacus. To that number add 183. Read me your answer.

$$\begin{array}{r} 548 \\ +183 \\ \hline 731 \end{array}$$

18

Good. or The correct answer is 731.

Clear your abacus. YES

There are four-digit numbers in this problem. Set 9,745 on your abacus. To that number add 6,372.

$$\begin{array}{r} 9{,}745 \\ +6{,}372 \\ \hline 16{,}117 \end{array}$$

19 The correct answer is 16,117. Was your answer correct?

Clear your abacus. YES

I'm going to read six rules of addition. To show that you know what number can be added with each rule set that number on your abacus.

	Clear 3, set 5	2
20	Clear 7, set 1 left	3
	Clear 9, set 1 left	1
	Clear 4, set 5	1
	Clear 8, set 1 left	2
	Clear 2, set 5	3

Good. or Restatement of rule indicating its application.

This problem is a little different but will give you a chance to use all the rules for adding 1, 2, and 3.

Set 1 on your abacus. Now add 2. This
time add 3. Your abacus should show that
1 + 2 + 3 = 6. Instead of clearing your
abacus add 1 + 2 + 3 again. Continue to add
21 1 + 2 + 3 until either 17, 18, or 19 is
recorded on your abacus.

Your final answer should be 18. Did you add correctly?

YES

You may clear your abacus.

Criterion Check

I'm going to read five problems which you should be able to work. I'll ask you to read your answer and will repeat a problem if you'd like.

28	389	4,739	99	488
+43	+237	+8,341	+36	+153
71	626	13,080	135	641
+85			+ 74	
156			209	

LESSON TWENTY-THREE

1	Let's begin this lesson by working a problem in which you can use all the rules of addition you've learned. Get ready to add. 9 + 3 + 9 + 4 + 5 + 84 + 7 + 33 + 6 - 160 + 4 + 2 + 2 + 2 + 9 + 1 + 4 + 1 Your final answer should be 25. Was your answer correct? Clear your abacus.	YES
2	Set 6 on your abacus. Can you use the rule clear 1, set 5 to add 4? In this lesson you'll learn a rule that will help you show that 6 + 4 = 10. The number 6 should be recorded on your abacus.	NO
3	Now, make your abacus say 10.	OBV
4	By clearing the 1 and the 5 you subtracted 6. What did you do with the ten-bead? Good. or You set the ten-bead.	ADDED IT OR SET IT
5	To subtract 6 and add 10 is a way of adding 4. We can say that clearing 6 and setting 1 left is a way of adding 4.	NONE
6	In this problem you'll need to use the rule clear 6, set 1 left to add 4. Set 26 on your abacus. To that number add 1 ten and 4 or 14. Forty is the sum of those two numbers. Was your answer correct? Clear your abacus.	26 +14 40 YES

7 This time let's use the rule clear 6, set 1 left to add 4 tens. Set 62 on your abacus. To that number add 4 tens and 5 or 45. Look above and below the bar when you read your answer.

$$\begin{array}{r} 62 \\ +45 \\ \hline 107 \end{array}$$

Good. or The correct answer is 107.

Clear your abacus.

---

8 Try this problem. As you work think about whether you're using the rule clear 6, set 1 left to add 4 in the ones, tens, hundreds, or thousands column. Set 6,528 on your abacus. To that number add 4,321.

$$\begin{array}{r} 6{,}528 \\ +4{,}321 \\ \hline 10{,}849 \end{array}$$

The correct sum is 10,849.

Did you use the rule in the ones, tens, hundreds, or thousands columns?

THOUSANDS

Good. or You should have used the rule in the thousands column.

Clear your abacus.

---

9 I'm going to read two statements. When I have finished, show me one or two fingers so that I will know which statement you think explains why we say set 1 left instead of saying set 10.

We could say set 10 because the rule can be used only if you're adding in the ones column. The tens column is to the left of the ones column.

ANSWER STARRED

*On an abacus each column to the left is worth ten times as much as the one before it. To say set 1 left means that the rule can be applied in any column on the abacus.

The last statement is the best explanation of why we say set 1 left.

Set 7 on your abacus. Will the rule clear 6, set 1 left help you show that 7 + 4 = 11? Try it before you answer. Did the rule work?

10

No comments. or The rule should have worked.

YES

Clear your abacus.

In this problem you'll need to use the rule clear 6, set 1 left twice. Set 76 on your abacus. To that number add 4 tens and 4 or 44. Read me your answer.

11

Good. or Your answer should be 120.

$$\begin{array}{r}76\\+44\\\hline 120\end{array}$$

Clear your abacus.

At the top of the page I have given you are three simple problems. Work each one on the abacus and circle the problem in which you used the rule clear 6, set 1 left to add 4.

12

10 + 4 = 14
4 + 4 = 8
* 8 + 4 = 12

ANSWER STARRED

You should have marked the third problem.

There are three more simple problems at the bottom of the sheet. This time circle the problem or problems in which you do not need to use the rule clear 6, set 1 left to add 4.

13

9 + 4 = 13
*5 + 4 = 9
16 + 4 = 20

ANSWER STARRED

You should have marked the second problem.

---

14 In this problem you should look for two chances to use the rule clear 6, set 1 left as a way of adding 4. Set 57 on your abacus. To that number add 24. Instead of clearing 81, add 42.

$$\begin{array}{r} 57 \\ +24 \\ \hline 81 \\ +42 \\ \hline 123 \end{array}$$

Your final answer should be 123. Did you get the correct answer?

Clear your abacus.

YES

Criterion Check

I'm going to read five problems which you should be able to work. When you're finished I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{rrrrr} 72 & 95 & 66 & 19 & 81 \\ +45 & +44 & +24 & +14 & +46 \\ \hline 117 & 139 & 90 & 33 & 127 \\ & & +41 & & +\ 14 \\ \hline & & 131 & & 141 \end{array}$$

---

15 One rule for adding 4 is clear 1, set 5. What is the other rule for adding 4?

CLEAR 6, SET 1 LEFT

Good. or Restatement of rule indicating its application.

---

16 You'll need to use both of the rules for adding 4 in this problem. Set 426 on your abacus. To that number add 444. Be sure you look above and below the bar when you read your answer.

$$\begin{array}{r} 426 \\ +444 \\ \hline 870 \end{array}$$

Good. or The correct answer is 870.

Clear your abacus.

---

17 Set 8 on your abacus. Now, add 4. Instead of clearing 12 add another 4. Continue to add by 4's until either 27, 28, or 29 is recorded on your abacus.

The number 28 should be recorded on your abacus. Did you add accurately?

Clear your abacus. YES

---

18 In this problem you'll need a rule for adding 4 and one other rule of addition which you've learned. Set 86 on your abacus. To that number add 74. Read me your answer.

I agree. or The correct answer is 160

Clear your abacus.

86
+74
160

Criterion Check

I'm going to read five problems which you should be able to work. I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

78	29	98	38	248
+47	+42	+14	+84	+164
125	71	112	122	412
	+46		+ 54	
	117		176	

---

19 To work this problem you'll need to use several rules of addition. Set 498 on your abacus. To that number add 364. Read me your answer

Good. or The correct answer is 862.

Clear your abacus.

498
+364
862

20 This problem has both addition and subtraction in it. Set 999 on your abacus. Then subtract 550 from it. Instead of clearing 449 add 454.

Your final answer should be 903. Was your answer correct?

Clear your abacus.

$$\begin{array}{r}999\\-550\\\hline 449\\+454\\\hline 903\end{array}$$

YES

21 There are four-digit numbers in this problem. You'll need to think carefully as you work. Set 7,391 on your abacus. To that number add 4,135.

Your final answer should be 11,526. Was your answer correct?

Clear your abacus.

$$\begin{array}{r}7{,}391\\4{,}135\\\hline 11{,}526\end{array}$$

YES

22 Set 4 on your abacus. Now, add 1. Add 2 and then add 3. This time add 4. The number 14 should be recorded on your abacus but do not clear it. Continue to add 1 + 2 + 3 + 4 until either 22, 24, or 26 is recorded on your abacus.

The number 24 should be set on your abacus. Did you add accurately?

Clear your abacus.

YES

Criterion Check

It's time for some problems which you should be able to work independently. I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{rrrrr}829 & 949 & 8{,}599 & 446 & 477\\+224 & +449 & +4{,}213 & +184 & +345\\\hline 1053 & 1398 & 12{,}812 & 630 & 822\\ & +\ 265 & & & \\\cline{2-2} & 1663 & & & \end{array}$$

LESSON TWENTY-FOUR

1 This is the beginning of an abacus lesson in which you'll learn a rule for adding 6 so set that number on your abacus. Now, just for fun, let's warm up with a problem that sounds almost like the count down for a rocket blast off. To the 6 which is recorded on your abacus add 5. Then add 4, 3, 2, 1 and 0 in that order.

Your final answer should be 21. Did you add correctly?

YES

Clear your abacus.

---

2 I'm going to read three phrases which explain how to add on the abacus. To show that you know what number can be added with each rule set that number on your abacus.

Clear 5, set 1 left	5
Clear 7, set 1 left	3
Clear 4, set 1 left	6

I agree. or Restatement of rule indicating its application.

---

3 This problem will give you a chance to use the rule for adding 6. Set 54 on your abacus. To that number add 26. Read me your answer.

$$\begin{array}{r}54\\+26\\\hline 80\end{array}$$

Good. or Eighty is the correct answer.

Clear your abacus.

---

4 Set the number 6 on your abacus. Can you show that 6 + 6 = 12 by clearing 4 and setting 1 left?

NO

There must be another way of adding 6.

5	The number 6 should still be recorded on your abacus. Add 1 to it. Is it possible to add 5 by setting an upper bead to the bar?	NO
6	Because it is not possible to add 5 directly, you may use the rule clear 5, set 1 left to help you add 5. Do it.	OBV
7	The number 12 should be recorded on your abacus. Add 6 so that your abacus will say 18. Did you need to use a special rule for adding 6?	NO
8	Now, let's add 6 to 18. First add 1, then use the rule that will help you add 5. The number 24 should be recorded on your abacus. We can summarize what you did to add 6 by saying that you set 1, cleared 5 and set 1 left. Clear your abacus.	OBV
9	Here's another problem in which you'll need to add 6 by setting 1, clearing 5 and setting 1 left. Set 16 on your abacus. To that number add 7 tens and 6 or 76. Ninety-two is the correct answer. Did you get the correct answer? Clear your abacus.	16 +76 92 YES
10	There are three simple problems on the sheet which I have given you. Work each problem on the abacus and then circle the one problem in which you used the rule set 1, clear 5, set 1 left. 4 + 6 = 10 1 + 6 = 7 *8 + 6 = 14 You should have circled the third problem.	ANSWER STARRED

This time I'm going to read three rules which explain how to add on the abacus. To show that you know what number can be added with each rule set that number on your abacus.

11

Clear 4, set 5	1
Clear 2, set 1 left	8
Set 1, clear 5, set 1 left	6

I agree. or Restatement of rule indicating its application.

Set 7 on your abacus. Perhaps the rule set 1, clear 5, set 1 left will help you show that 7 + 6 = 13. Try it. Did the rule work?

12

No comment. or The rule should work.

Clear your abacus so you'll be ready for another problem. YES

Try this problem. Set 78 on your abacus. To that number add 6 tens and 6 or 66. Read me your answer.

13

Good. or Your final answer should be 144.

$$\begin{array}{r} 78 \\ +66 \\ \hline 144 \end{array}$$

Clear your abacus.

Here's a problem in which you'll need to use the rule for adding 6 several times. Set 57 on your abacus. Now, add 16. Instead of clearing 73 add 65.

14

$$\begin{array}{r} 57 \\ +16 \\ \hline 73 \\ +65 \\ \hline 138 \end{array}$$

Your final answer should be 138. Was your answer correct?

Clear your abacus. YES

Criterion Check

I'm going to read five problems which you should be able to work. When you've finished, I'll ask you to read your answer aloud.

$$\begin{array}{r} 18\\ +16\\ \hline 34 \end{array} \qquad \begin{array}{r} 56\\ +16\\ \hline 72\\ +65\\ \hline 137 \end{array} \qquad \begin{array}{r} 81\\ +66\\ \hline 147 \end{array} \qquad \begin{array}{r} 76\\ +66\\ \hline 142 \end{array} \qquad \begin{array}{r} 55\\ +63\\ \hline 118\\ +\ 76\\ \hline 194 \end{array}$$

---

15 I have given you a sheet which has three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used the rule set 1, clear 5, set 1 left to add 6.

*72 + 61 = 133
*18 + 66 = 84
41 + 56 = 97

You should have marked the first two problems.

ANSWERS STARRED

---

16 There are two rules for adding 6. One rule says clear 4, set 1 left. What is the other rule for adding 6?

Good. or To set 1, clear 5, set 1 left is a way of adding 6.

SET 1, CLEAR 5, SET 1 LEFT

---

17 Here's a problem in which you can use both of the rules for adding 6. Begin by setting 75 on your abacus. Then add 16. Finally add 62.

Your final answer should be 153. Did you add correctly?

Clear your abacus.

$$\begin{array}{r} 75\\ +16\\ \hline 91\\ +62\\ \hline 153 \end{array}$$

YES

---

18 Set 7 on your abacus. Add 6. Continue to add by 6 until either 40, 43, or 45 is recorded on your abacus. Read me your final answer.

Good. or Forty-three is the correct answer.

clear your abacus.

---

19 In this problem you'll need to use the rules for adding 6 and one other rule of addition. Set 44 on your abacus. To that number add 62. Then add 86.

$$\begin{array}{r} 44 \\ +62 \\ \hline 106 \\ +\ 86 \\ \hline 192 \end{array}$$

Your final answer should be 192. Did you add correctly?

Clear your abacus.

YES

Criterion Check

I'm going to read five problems which you should be able to work. Raise your hand if you'd like me to repeat a problem. I'll ask you to read your results aloud.

$$\begin{array}{r} 819 \\ +666 \\ \hline 1485 \end{array} \qquad \begin{array}{r} 98 \\ +\ 6 \\ \hline 104 \\ +\ 96 \\ \hline 200 \end{array} \qquad \begin{array}{r} 25 \\ +16 \\ \hline 41 \\ +62 \\ \hline 103 \end{array} \qquad \begin{array}{r} 59 \\ +\ 6 \\ \hline 65 \\ +26 \\ \hline 91 \end{array} \qquad \begin{array}{r} 17 \\ +26 \\ \hline 43 \\ +65 \\ \hline 108 \end{array}$$

---

20 You'll need to use several rules of addition in this problem which has three-digit numbers in it. Set 289 on your abacus. To that number add 236. Look above and below the bar when you read your answer.

$$\begin{array}{r} 289 \\ +236 \\ \hline 525 \end{array}$$

Good. or The correct answer is 525.

Clear your abacus.

I'm going to read three rules which explain how to add on the abacus. To show that you know what number can be added with each rule, record that number on your abacus.

21

Clear 5, set 1 left	5
Clear 6, set 1 left	4
Set 1, clear 5, set 1 left	6

Good. or Restatement of rule indicating its application.

---

22 Try this problem. Set 86 on your abacus. To that number add 63. Now add 65.

Your final answer should be 214. Did you add correctly?

Clear your abacus.

$$\begin{array}{r} 86 \\ +63 \\ \hline 149 \\ +\ 65 \\ \hline 214 \end{array}$$

YES

---

23 There are four-digit numbers in this problem. Set 6,881 on your abacus. To that number add 6,143. There should be five digits in your answer so read it carefully.

Good. or The correct answer is 13,024.

Clear your abacus.

$$\begin{array}{r} 6{,}881 \\ +6{,}143 \\ \hline 13{,}024 \end{array}$$

---

24 Here's another problem that has four-digit numbers in it. Begin by setting 1,828 on your abacus. To that number add 6,654. Look above and below the bar when you read your answer.

I agree. or Your answer should be 8,482.

Clear your abacus.

$$\begin{array}{r} 1828 \\ +6654 \\ \hline 8482 \end{array}$$

The final problem in this lesson is one in which you can use every rule you know. I'll read slowly so that you will have time to work accurately. 4 + 3 + 6 + 4 + 8 + 5 +79 + 2 + 9 - 120 + 9 + 6 + 4 + 1 + 9 + 3
25 + 22 + 1 - 55 + 8 + 4

Your final answer should be 12. Was your answer correct?

YES

Clear your abacus.

Criterion Check

I'm going to read five problems. When you have finished I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{r} 589 \\ \underline{+642} \\ 1231 \\ \\ \\ \end{array} \qquad \begin{array}{r} 829 \\ \underline{+531} \\ 1360 \\ \underline{+\ 856} \\ 2216 \end{array} \qquad \begin{array}{r} 779 \\ \underline{+631} \\ 1410 \\ \\ \\ \end{array} \qquad \begin{array}{r} 428 \\ \underline{+516} \\ 944 \\ \underline{+216} \\ 1160 \end{array} \qquad \begin{array}{r} 6,685 \\ \underline{+6,453} \\ 13,138 \\ \\ \\ \end{array}$$

LESSON TWENTY-FIVE

1	You've learned fourteen of the special rules that will help you add on the abacus. To work this problem you will need to use nearly all of those rules. 6 + 6 + 96 + 24 + 87 + 23 + 77 + 21 + 69 + 33 Your final answer should be 442. Was your answer correct? Clear your abacus.	YES
2	Set 7 on your abacus. Is it possible to show that 7 + 7 = 14 by clearing 3 and setting 1 left? There must be another way of adding 7.	NO
3	The number 7 should be recorded on your abacus. To show that 7 + 7 = 14 you would need to add 2 and then 5. First add 2. Is it possible to add 5 by setting an upper bead to the bar?	NO
4	Since the five-bead has already been used, you'll need to use the rule for adding 5. Do it.	OBV
5	We can summarize what you did to add 7 by saying that you set 2, cleared 5 and set 1 left. Clear 14.	NONE
6	Let's work a problem in which you can add 7 by setting 2, clearing 5 and setting 1 left. Set 26 on your abacus. To that number add 5 tens and 7 or 57. Eighty-three is the correct answer. Did you get the correct answer? Clear your abacus.	$\begin{array}{r} 26 \\ +57 \\ \hline 83 \end{array}$ YES

7 You should use the rule set 2, clear 5, set 1 left to work this problem. Set 171 on your abacus. To that number add 576. Look above and below the bar when you read your answer.

171
+576
747

Good. or Your final answer should be 747.

Clear your abacus.

---

8 I've given you a sheet with three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used the rule set 2, clear 5, set 1 left to add 7.

71 + 7 = 78
13 + 17 = 30
*17 + 7 = 24

ANSWER STARRED

You should have circled the third problem.

---

9 In this problem you'll need to use the rule set 2, clear 5, set 1 left twice. Set 657 on your abacus. To that number add 277. Read me your answer.

657
+277
934

Good. or Your final answer should be 934.

Clear your abacus.

Criterion Check

It's time for five problems which you should be able to work independently. When you've finished I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

25	66	557	71	265
+17	+72	+717	+75	+177
42	138	1274	146	442

10	There are two rules for adding 7. One rule says clear 3, set 1 left. What is the other rule for adding 7. Good. or To set 2, clear 5, set 1 left is a way of adding 7.	SET 2, CLEAR 5, SET 1 LEFT
11	Here is a problem in which you can use both rules for adding 7. Set 195 on your abacus. To that number add 277. Read me your answer. Good. or Your final answer should be 472. Clear your abacus.	195 +277 472
12	I have given you a sheet with three problems on it. The correct answers have been given. Work each problem for youself and then circle the problem or problems in which you used the rule clear 3, set 1 left to add 7. 66 + 73 = 139 52 + 76 = 128 *28 + 67 = 95 You should have circled the third problem.	ANSWER STARRED
13	Try this problem. Set 568 on your abacus. To that number add 777. The correct answer is 1,345. Did you get the correct answer? Clear your abacus.	568 +777 1345 YES
14	Set 9 on your abacus. Then add 7 so that your abacus will say 16. Continue to add by 7's until either 42, 43, or 44 appears on your abacus. What number is recorded on your abacus? Good. or Your final answer should be 44. Clear your abacus.	44

Criterion Check

I'm going to read five problems which you should be able to work. Then I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{r} 96 \\ +77 \\ \hline 173 \end{array} \qquad \begin{array}{r} 16 \\ +\ 7 \\ \hline 23 \\ +17 \\ \hline 40 \end{array} \qquad \begin{array}{r} 514 \\ +737 \\ \hline 1251 \end{array} \qquad \begin{array}{r} 8 \\ +57 \\ \hline 65 \\ +72 \\ \hline 137 \end{array} \qquad \begin{array}{r} 351 \\ +777 \\ \hline 1128 \end{array}$$

15 In this problem you'll need to use the rule for adding 7 and one other rule of addition. Set 75 on your abacus. To that number add 4 tens and 7 or 47. Read me your answer.

$$\begin{array}{r} 75 \\ +47 \\ \hline 122 \end{array}$$

Good. or The correct answer is 122.

Clear your abacus.

16 In this problem you'll need to use a rule for adding 1 to help you set 1 left. Begin by setting 97 on your abacus. To that number add 7. Instead of clearing 104 add 28.

$$\begin{array}{r} 97 \\ +\ 7 \\ \hline 104 \\ +\ 28 \\ \hline 132 \end{array}$$

Your final answer should be 132. Was your answer correct?

Clear your abacus. YES

17 There are three-digit numbers in this problem. Set 674 on your abacus. To that number add 765. Look above and below the bar when you read your answer.

$$\begin{array}{r} 674 \\ +765 \\ \hline 1439 \end{array}$$

I agree. or The correct answer is 1,439.

Clear your abacus.

Try this problem which has four-digit numbers in it. Set 9,679 on your abacus. To that number add 7,742. There are five digits in your answer so read it carefully.

18

Good. or Your answer should be 17,421.

Clear your abacus.

9,679
+7,742
17,421

This problem has both addition and subtraction in it. Set 1,835 on your abacus. To that number add 2,647. Now, subtract 3,161.

19

Your final answer should be 1,321. Was your answer correct?

Clear your abacus.

1835
+2647
4482
-3161
1321

YES

Before you work some problems independently, try this fun problem as a way of reviewing what you've learned. What is the sum of 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8?

20 36

Your final answer should be 36. Did you add correctly?

Clear your abacus. YES

Criterion Check

I'm going to read five problems which you should be able to work. As usual I'll ask you to read your answer aloud and will repeat a problem if necessary.

666	71	5,899	8,725	53
+574	+74	+7,913	+8,766	+78
1240	145	13,812	17,491	131
	+ 56			+ 36
	201			167

LESSON TWENTY-SIX

To begin this lesson and review what you've learned let's work a problem which will give you an opportunity to use all the rules you've learned. I'll read slowly so that you'll have plenty of time to work accurately.
6 + 7 + 8 + 48 + 31 + 8 + 6 + 52 + 43 + 23
1 + 51 -230 + 7.

Your final answer should be 60. Was your answer correct?

YES

Clear Your abacus.

In the last two lessons you learned rules to help you add 6 and 7. What number do you
2 think you'll learn to add in this lesson? 8

Set 8 on your abacus.

Add 8 to 8 so that your abacus will say 16. What rule did you use to show that 8 + 8 = 16?

3 CLEAR 2, SET 1 LEFT

Good. or Restatement of rule indicating its application.

Clear your abacus.

To clear 5 and set 1 left is a way of adding 5. Adding five is part of some other rules of addition.

4 I've given you a sheet on which there are three statements which explain how to add on the abacus. Read the statements silently as I read them aloud. Draw a circle around each sentence in which the rule for adding five is part of the statement. (Reader, emphasize the words and phrases that have been underlined.)

ANSWERS STARRED

*Set 2, clear 5, set 1 left is a way of adding 7.
Clear 2, set 1 left is a way of adding 8.
*Set 1, clear 5, set 1 left is a way of adding 6.

The rule for adding five is a part of the rules for adding six and seven which I read.

5	Set 5 on your abacus. Can you use the rule clear 2, set 1 left to show that 5 + 8 = 13? There must be another way of adding 8. The number 5 should still be recorded on your abacus.	NO
6	You know that 3 + 5 = 8. Add 8 to the 5 which is recorded on your abacus. You may need to use the rule clear 5, set 1 left to help you.	OBV
7	We can summarize what you did to add 8 by saying that you set 3, cleared 5, and set 1 left. Clear 13 from your abacus.	OBV
8	Here's a problem in which you can use the rule set 3, clear 5, set 1 left to add 8. Set 55 on your abacus. To that number add 2 tens and 8 or 28. Read me your answer. Good. or The correct answer is 83. Use your index finger to clear your abacus.	55 +28 83
9	Set 6 on your abacus. Perhaps the rule set 3, clear 5, set 1 left will help you show that 6 + 8 = 14. Try it. Did the rule work? No comment. or The rule should work. Clear your abacus.	YES
10	This problem will give you two chances to add 8 by setting 3, clearing 5 and setting 1 left. Set 56 on your abacus. To that number add 8 tens and 8 or 88. Read me your answer. I agree. or The correct answer is 144. Clear your abacus.	56 +88 144

11	I'm going to read three phrases. When I have finished set 1, 2, or 3 on your abacus so that I will know how many of the phrases you think explain a way of adding eight. *Clear 2, set 1 left *Set 3, set 5 *Set 3, clear 5, set 1 left All three phrases explain a way of adding eight.	ANSWERS STARRED
12	Try this problem. Set 45 on your abacus. To that number add 8 tens and 8 or 88. Read me your answer. Good. or The correct answer is 133. Clear your abacus.	45 +88 133
13	Set 4 on your abacus. Then add 8 to it. Continue to add by 8's until either 44, 45, or 49 is recorded on your abacus. Your final answer should be 44. You may clear your abacus.	OBV
14	There are three-digit numbers in this problem. Set 351 on your abacus. To that number add 888. Look above and below the bar when you read your answer. Good. or The correct answer is 1,239. Clear your abacus.	351 +888 1239

Criterion Check

I'm going to read seven problems which you should be able to work. I'll ask you to read your answer aloud and will repeat a problem if necessary.

65	36	53	736	56	96	55
+28	+88	+ 8	+118	+38	+88	+38
93	124	61	854	94	184	93
		+85				+85
		146				178

I'm going to read four statements about addition on the abacus. Raise your hand if the statement is true. If the statement is not true, sit quietly.

The difference between 6 and 10 is 4 so it is possible to add 4 by clearing 6 and setting 10.

15 Because 3 + 5 = 8 you may add 8 by setting 3 and then using the rule that is a way of adding 5.

The rules of abacus addition can be used in any column on the abacus.

It is possible to add 6 by setting 1, clearing 5 and setting 1 left.

All of the statements are true.

To begin this problem set 58 on your abacus. Then add 84. Finally add 81.

16 The correct answer is 223. Did you add correctly?

Clear your abacus.

58
+84
142
+ 81
223
YES

---

17 Try this problem. Set 65 on your abacus. To that number add 6 tens and 8 or 68. Now add 58.

Your final answer should be 191. Did you add accurately?

Clear your abacus.

$$\begin{array}{r} 65 \\ +68 \\ \hline 133 \\ +\ 58 \\ \hline 191 \end{array}$$

YES

18 This problem will give you a chance to use several rules of addition. Set 64 on your abacus. To that number add 83. Instead of clearing your abacus add 27.

Your final answer should be 174. Did you add correctly?

Clear your abacus.

$$\begin{array}{r} 64 \\ +83 \\ \hline 147 \\ +\ 27 \\ \hline 174 \end{array}$$

YES

19 This time let's work a problem in which there are three-digit numbers. Set 257 on your abacus. To that number add 787. Read me your answer.

Good. or The correct answer is 1,044.

Clear your abacus.

$$\begin{array}{r} 257 \\ +787 \\ \hline 1044 \end{array}$$

20 This time let's try a problem that has both addition and subtraction in it. Begin by setting 496 on your abacus. To that number add 718. Now, subtract 1,202.

Your final answer should be 12. Did you work accurately?

Clear your abacus.

$$\begin{array}{r} 496 \\ +718 \\ \hline 1214 \\ -1202 \\ \hline 12 \end{array}$$

YES

21 This problem has both addition and subtraction in it. Set 2,468 on your abacus. To that number add 4 thousands, 3 hundreds, 8 tens and 1. Then subtract 1,345. Be sure you look above and below the bar when you read your answer.

$$\begin{array}{r} 2468 \\ +4381 \\ \hline 6849 \\ -1345 \\ \hline 5504 \end{array}$$

I agree. or The correct answer is 5,504.

Clear your abacus.

Criterion Check

I'm going to read eight problems which you should be able to work. When you've finished I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{r} 976 \\ +168 \\ \hline 1144 \end{array} \quad \begin{array}{r} 46 \\ +26 \\ \hline 72 \\ +36 \\ \hline 108 \end{array} \quad \begin{array}{r} 97 \\ +67 \\ \hline 164 \\ +\ 74 \\ \hline 238 \end{array} \quad \begin{array}{r} 369 \\ +952 \\ \hline 1321 \end{array} \quad \begin{array}{r} 7,676 \\ +6,588 \\ \hline 14,264 \end{array} \quad \begin{array}{r} 736 \\ +378 \\ \hline 1114 \end{array} \quad \begin{array}{r} 52 \\ +34 \\ \hline 86 \\ +88 \\ \hline 174 \end{array} \quad \begin{array}{r} 58 \\ +84 \\ \hline 142 \\ +\ 68 \\ \hline 210 \end{array}$$

LESSON TWENTY-SEVEN

1 You've practiced all the rules of abacus addition except one. In this lesson you'll practice using that rule. Before we begin something new, try this problem which will give you a chance to use everything you've learned. I'll read slowly so that you'll have plenty of time to work accurately.
5 + 8 + 7 + 39 + 21 + 26 + 65 + 71 + 91 + 8

Your final answer should be 341. Did you add accurately?

Clear your abacus. YES

---

2 You know that 4 + 5 = 9. In some cases it is possible to record 9 by setting four lower beads and then a five-bead to the bar. Set 10 on your abacus and then add 9 to it. OBV

Clear 19.

---

3 In other cases you must set four lower beads and then use the rule for 5 to help you record that part of 9. Set 5 on your abacus. Now, add 9 to it. OBV

---

4 We can summarize what you did to add 9 by saying that you set 4, cleared 5 and set 1 left. The rule for adding 5 is part of the rule for adding 6, 7, and 8. It is also a part of the rule for adding 9. NONE

Clear 14 from your abacus.

---

5 Here's a problem in which you can use the rule set 4, clear 5, set 1 left to add 9. Set 35 on your abacus. To that number add 59. Read me your answer.

$$\begin{array}{r}35\\+59\\\hline 94\end{array}$$

Good. or The correct answer is 94.

Clear your abacus.

---

6 There are three-digit numbers in this problem. Set 215 on your abacus. To that number add 529. Look above and below the bar when you read your answer.

I agree. or The correct answer is 744.

Clear your abacus.

$$\begin{array}{r} 215 \\ +529 \\ \hline 744 \end{array}$$

7 I'm going to read three phrases which explain a way of adding on the abacus. Raise your hand each time you hear a phrase which explains a way of adding 9.

*Clear 1, set 1 left
*Set 4, set 5
*Set 4, clear 5, set 1 left

All three phrases explain a way of adding 9.

ANSWERS STARRED

8 Try this problem. Set 85 on your abacus. To that number add 99.

Your answer should be 184. Did you add accurately?

Clear your abacus.

$$\begin{array}{r} 85 \\ +99 \\ \hline 184 \end{array}$$

YES

9 I have given you a sheet which has three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you added 9 by setting 4, clearing 5, and setting 1 left.

29 + 19 = 48
*15 + 29 = 44
30 + 9 = 39

You should have circled the second problem.

ANSWER STARRED

Begin this problem by setting 805 on your abacus. To that number add 999.

10 The correct answer is 1,804. Was your answer correct?

805
+999
1804

Clear your abacus.

YES

Set 5 on your abacus. Add 9 so that your abacus will say 14. Continue to add by nines until the number 100, 102 or 104 is recorded on your abacus.

11 Your final answer should be 104.

OBV

You may clear your abacus.

Criterion Check

I'm going to read seven problems which you should be able to work. As usual I will ask you to read your results aloud.

75	53	55	353	55	425	15
+19	+91	+29	+995	+39	+909	+19
94	144	84	1348	94	1334	34
		+ 9				+59
		93				93

Let's add several numbers this time. You'll need to use more than one rule of addition. Set 54 on your abacus. To that number add 31. Now add 29.

12 Your final answer should be 114. Did you add correctly?

54
+31
85
+29
114

Clear your abacus.

13

This problem is similar to the last one. Set 64 on your abacus. To that number add 8 tens and 7 or 87. Now add 95. Look above and below the bar when you read your answer.

I agree. or Your final answer should be 246.

$$\begin{array}{r} 64 \\ +87 \\ \hline 151 \\ +\ 95 \\ \hline 246 \end{array}$$

Clear your abacus.

14

There are three-digit numbers in this problem. Set 395 on your abacus. To that number add 5 hundreds, 6 tens and 9 or 569. Read me your answer.

Good. or The correct answer is 964.

$$\begin{array}{r} 395 \\ +569 \\ \hline 964 \end{array}$$

Clear your abacus.

15

This problem has both addition and subtraction in it. Set 155 on your abacus. To that number add 259. Now subtract 202.

Your final answer should be 212. Was your answer correct?

$$\begin{array}{r} 155 \\ +259 \\ \hline 414 \\ -202 \\ \hline 212 \end{array}$$

Clear your abacus.

16

In this problem you'll need to add three three-digit numbers. Set 342 on your abacus. To that number add 353. Now add 639.

Your final answer should be 1,334. Was your answer correct?

$$\begin{array}{r} 342 \\ +353 \\ \hline 695 \\ +639 \\ \hline 1334 \end{array}$$

Clear your abacus.

---

17 There are four-digit numbers in this problem. Set 4,465 on your abacus. To that number add 1,749. Read me your answer.

I agree. or The correct answer is 6,214.

Clear your abacus.

4465
+1749
6214

---

18 Begin this problem by setting 5,226 on your abacus. Then add 9,258, that is 9 thousands, 2 hundreds, 5 tens and 8.

Your final answer should be 14,484. Did you work accurately?

Clear your abacus.

5,226
+9,258
14,484

Criterion Check

I'm going to read eight problems which you should be able to work. As usual I'll ask you to read your results aloud and will repeat a problem if necessary.

65	455	25	9,756	55	595	89	5,439
+79	+289	+29	+1,398	+69	+879	+42	+9,871
144	744	54	11,154	124	1474	131	15,310
+ 83		+53		+ 82		+ 79	
227		107		206		210	
		+ 24				+ 99	
		131				309	

LESSON TWENTY-EIGHT

1	You've learned all the rules necessary for addition on the abacus. You've done a few subtraction problems but have not used any special rules to help you subtract. In this lesson you'll begin to learn the special rules of subtraction.	NONE
2	Set 5 on your abacus. Now make your abacus say 4. What number did you subtract when you set 4 and cleared 5?	1
3	Let's check to be sure that setting 4 and clearing 5 is a way of subtracting 1. Set 25 on your abacus. Now use the rule set 4, clear 5 to subtract 1 from 25. The number 24 should appear on your abacus. You may clear your abacus.	OBV
4	To set 4 and clear 5 is a way of subtracting 1. You should set 4 with your thumb and clear 5 with your index finger.	NONE
5	Here's a problem in which you can use the rule set 4, clear 5 to subtract 1. Set 35 on your abacus. From that number subtract 2 tens and 1 or 21. Your final answer should be 14. Did you subtract accurately? Clear your abacus.	35 -41 14 YES
6	This problem will give you more than one chance to use the rule for subtracting 1. Set 85 on your abacus. From that number subtract 3 tens and 1 or 31. The number 54 should appear on your abacus. Now subtract 13. Your final answer should be 41. Did you subtract accurately? Clear your abacus.	85 -31 54 -13 41 YES

This problem is similar to the last one. Set 59 on your abacus. From that number subtract 1 ten and 4 or 14. Then subtract 41. Read me your answer.

$$\begin{array}{r} 59 \\ -14 \\ \hline 45 \\ -41 \\ \hline 4 \end{array}$$

7

Good. or Four is the correct answer.

Use your index finger to clear your abacus.

I'm going to read three phrases which describe a way of calculating on the abacus. Raise your hand when you hear a phrase which explains how to subtract 1.

ANSWER STARRED

8

Set 1
Clear 1, set 5
*Set 4, clear 5

To set 4 and clear 5 is a way of subtracting 1.

It is possible to use a rule of addition in any column on the abacus. The rules of subtraction may also be applied in the ones, tens, hundreds, or thousands column.

9 As you work this problem think about whether you are applying the rule for subtracting 1 in the ones column, the tens column, or the hundreds column. Set 549 on your abacus. From that number subtract 131. To show a difference of 418 did you apply the rule in the ones column, the tens column, or the hundreds column?

$$\begin{array}{r} 549 \\ -131 \\ \hline 418 \end{array}$$

HUNDREDS

I agree. or You should have used the rule in the hundreds column.

Clear your abacus.

There are four-digit numbers in this problem. Set 2,457 on your abacus. From that number subtract 1,216. Read me your answer.

10

Good. or The correct answer is 1,241.

Clear your abacus.

2457
-1216
1241

This time begin by setting 45,999 on your abacus. From that very large number subtract 21,383.

11

Your final answer should be 24,616. Was your answer correct?

Clear your abacus.

45,999
-21,383
24,616

YES

Criterion Check

I'm going to read seven problems. Raise your hand if you'd like me to repeat a problem. As usual I'll ask you to read your answer aloud.

95	58	65	465	58	75	56
-41	-13	-11	-111	-12	-21	-11
54	45	54	354	46	54	45
	-21	-12			-11	-21
	24	42			43	24

This problem has four-digit numbers in it. Begin by setting 5,958 on your abacus. From that number subtract 1,717.

12

Your final answer should be 4,241. Did you subtract accurately?

Clear your abacus.

5958
-1717
4241

YES

13 There are three-digit numbers in this problem. Set 599 on your abacus. From that number subtract 1 hundred, 8 tens and 5. Now subtract 202. Read me your answer.

I agree. or Your final answer should be 212.

Clear your abacus.

$$\begin{array}{r} 599\\ -185\\ \hline 414\\ -202\\ \hline 212 \end{array}$$

---

14 Here's another problem which has three- and four-digit numbers in it. Set 1,956 on your abacus. From that number subtract 611. Instead of clearing your abacus subtract 121. Read me your answer.

Good. or The correct answer is 1,224.

Clear 1,224.

$$\begin{array}{r} 1956\\ -\ 611\\ \hline 1345\\ -\ 121\\ \hline 1224 \end{array}$$

Criterion Check

I'm going to read eight problems which you should be able to work. I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{rrrrrrrr} 85 & 59 & 568 & 5713 & 95 & 57 & 582 & 4385\\ -31 & -14 & -112 & -1202 & -41 & -12 & -130 & -2231\\ \hline 54 & 45 & 456 & 4511 & 54 & 45 & 452 & 2154\\ -13 & -31 & -211 & -1111 & -10 & -21 & -211 & -1111\\ \hline 41 & 14 & 245 & 3400 & 44 & 24 & 241 & 1043 \end{array}$$

LESSON TWENTY-NINE*

1	You've begun to learn how to subtract on the abacus. In the last lesson you learned a rule for subtracting 1. Let's work a problem in which you can practice that rule before beginning something new. Set 659 on your abacus. From that number subtract 515. Read me your answer. Good. or The correct answer is 144. Clear your abacus.	659 -515 144 OBV
2	Set 5 on your abacus. Now make your abacus say 3. You know that the difference between 5 and 3 is 2. To set 3 and clear 5 is a way of subtracting 2.	NONE
3	A rule for subtracting 2 is set 3 and clear 5. Show me the finger you should use to set 3. No comment. or You should use your right thumb. Show me the finger you should use to clear 5. No comment. or You should use your right index finger.	RIGHT THUMB RIGHT INDEX
4	Here's a problem in which you can use the rule set 3, clear 5 to subtract 2. Set 45 on your abacus. From that number subtract 32. The difference is 13. Did you subtract correctly? Clear your abacus.	45 -32 13 YES
5	Set 8 on your abacus. Now subtract 2. Did you use the rule set 3, clear 5 to show that 8 - 2 = 6? Do not clear 6.	NO

6	Subtract 2 from 6. Did you use the rule set 3, clear 5 to show that 6 - 2 = 4? Clear your abacus.	YES
7	As you work this problem look for a chance to use the rule set 3, clear 5 as a way of subtracting 2. Set 86 on your abacus. From that number subtract 5 tens and 2 or 52. Read me your answer. I agree. or The correct answer is 34. Clear your abacus.	86 -52 34
8	At the top of the page I have given your three problems. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you use the rule set 3, clear 5 to subtract 2. 79 - 22 = 57 *86 - 62 = 24 48 - 26 = 22 You should have marked the second problem.	ANSWER STARRED
9	There are also three problems at the bottom of the sheet. The correct answers have been given. This time circle the one problem in which you do not need to use the rule set 3, clear 5 as a way of subtracting 2. 68 - 21 = 47 65 - 22 = 43 *47 - 26 = 21 You should have marked the third problem.	ANSWER STARRED
10	What is the rule for subtracting 1 on an abacus? No comment. or To set 4 and clear 5 is a way of subtracting 1.	SET 4, CLEAR 5

11 In this problem you'll need to use the rule for subtracting 1 and the rule for subtracting 2. Set 59 on your abacus. From that number subtract 14. Now subtract 42.

Your final answer should be 3. Did you subtract correctly?

Clear your abacus.

$$\begin{array}{r} 59 \\ \underline{-14} \\ 45 \\ \underline{-42} \\ 3 \end{array}$$

YES

Criterion Check

I'm going to read seven problems which you should be able to work. Raise your hand if you would like me to repeat a problem. I'll ask you to read your answer aloud.

$$\begin{array}{rrrrrrr} 85 & 66 & 58 & 85 & 96 & 59 & 95 \\ \underline{-52} & \underline{-25} & \underline{-23} & \underline{-21} & \underline{-72} & \underline{-24} & \underline{-31} \\ 33 & 41 & 35 & 64 & 24 & 35 & 64 \\ & & \underline{-21} & \underline{-24} & & & \underline{-23} \\ & & 14 & 40 & & & 41 \end{array}$$

---

12 There are three-digit numbers in this problem. Set 555 on your abacus. From that number subtract 521. Read me your answer.

I agree. or Thirty-four is the correct answer.

Use your index finger to clear your abacus.

$$\begin{array}{r} 555 \\ \underline{-521} \\ 34 \end{array}$$

---

13 Try this subtraction problem. Set 958 on your abacus. From that number subtract 4 hundreds, 1 ten and 2 or 412. Now subtract 232.

Your final answer should be 314. Was your answer correct?

Clear your abacus.

$$\begin{array}{r} 958 \\ \underline{-412} \\ 546 \\ \underline{-232} \\ 314 \end{array}$$

YES

14 There are four-digit numbers in this problem. Set 4,567 on your abacus. From that number subtract 3,225. Read me your answer.

Fine. or The correct answer is 1,342.

Clear your abacus.

$$\begin{array}{r} 4567 \\ -3225 \\ \hline 1342 \end{array}$$

---

15 This problem has several numbers in it. Begin by setting 7,985 on your abacus. From that number subtract 1,452. Now, subtract 2,121.

Your final answer should be 4,412. Did you subtract accurately?

$$\begin{array}{r} 7985 \\ -1452 \\ \hline 6533 \\ -2121 \\ \hline 4412 \\ \text{YES} \end{array}$$

Criterion Check

I'm going to read eight problems. As usual I'll ask you to read your results aloud and will repeat a problem if necessary.

$$\begin{array}{rrrrrrrr} 156 & 69 & 695 & 555 & 567 & 58 & 58 & 564 \\ -\ 12 & -24 & -281 & -221 & -121 & -12 & -23 & -123 \\ \hline 144 & 45 & 414 & 334 & 446 & 46 & 35 & 441 \\ -123 & -21 & & & -332 & -12 & -21 & \\ \cline{1-2}\cline{5-7} 21 & 24 & & & 114 & 34 & 14 & \end{array}$$

LESSON THIRTY

1	Let's begin this lesson by working a problem in which you can use what you've already learned about subtraction. Set 76 on your abacus. From that number subtract 22. Now subtract 13. Your final answer should be 41. Did you subtract correctly? Clear your abacus.	76 -22 54 -13 41 YES
2	So far you've learned rules for subtracting 1 and 2. What number do you think you'll learn to subtract in this lesson?	3
3	Set 5 on your abacus. You know that 5 - 3 = 2. Make your abacus say 2. To subtract 3 from 5 you set 2 lower beads and cleared the 5 bead. Clear your abacus.	OBV
4	This time set 6 on your abacus. Can you show that 6 - 3 = 3 by clearing 3 lower beads?	NO
5	You found that setting 2 and clearing 5 was a way of subtracting 3 from 5. Try setting 2 and clearing 5 as a way of subtracting 3 from 6. Is the number 3 recorded on your abacus? That way of subtracting 3 must be a good one.	YES
6	This time set 7 on your abacus. Can you subtract 3 from 7 by clearing three lower beads from the bar?	NO

7

Setting 2 and clearing 5 has shown that 5 - 3 = 2 and that 6 - 3 = 3. Try that way of subtracting 3 from 7. Is the rule a good one to use in that case?

No comment. or The rule should work.

Clear your abacus.

YES

8

Here's a problem in which you can use the rule set 2, clear 5 as a way of subtracting 3. Set 96 on your abacus. From that number subtract 7 tens and 3 or 73. Read me your answer.

$$\begin{array}{r} 96 \\ -73 \\ \hline 23 \end{array}$$

Good. or Twenty-three is the correct answer.

Clear your abacus.

9

Try this problem. It will give you two chances to use the rule set 2, clear 5. Begin by setting 85 on your abacus. From that number subtract 1 ten and 3 or 13. Now subtract 31.

$$\begin{array}{r} 85 \\ -13 \\ \hline 72 \\ -31 \\ \hline 41 \end{array}$$

Your final answer should be 41. Did you get the correct answer?

Clear your abacus.

YES

10

I have given you a sheet with several problems on it. The correct answers have been given. Find the three problems at the top of the page. Work each problem for yourself and then circle the problem or problems in which you used the rule set 2, clear 5 to subtract 3.

ANSWERS STARRED

493 - 263 = 230
*64 - 32 = 32
*75 - 33 = 42

You should have used the rule set 2, clear 5 to subtract 3 in the last two problems.

11 Find the three problems on the bottom of the page. The correct answers for these problems have been given. This time circle the problem or problems in which you do not need to use the rule set 2, clear 5 to subtract 3.

25 - 13 = 12
*86 - 35 = 51
76 - 31 = 45

ANSWER STARRED

You should have circled the second problem.

Criterion Check

I'm going to read five problems which you should be able to work. As usual I will ask you to read your answers aloud. Raise your hand if you would like me to repeat a problem.

95	36	79	547	67
-63	-23	-33	-333	-53
32	13	46	214	14
		-23		
		23		

---

12 Set 66 on your abacus. From that number subtract 12. Your answer should be 54. What rule did you use when you subtracted the 2 ones of 12?

SET 3, CLEAR 5

I agree. or You set 3 and cleared 5.

---

13 I'm going to read three statements. Raise your hand each time you hear a true statement. If a statement is not true, you should sit quietly.

The difference between 3 and 5 is 2, so you can subtract 2 by setting 3 and clearing 5.

The difference between 4 and 5 is 1, so you can subtract 1 by setting 4 and clearing .

The difference between 2 and 5 is 3, so you can subtract 3 by setting 2 and clearing 5.

All of the statements were true.

14	In this problem you'll need to use more than one rule of subtraction. Set 59 on your abacus. From that number subtract 12. Now subtract 23. Your final answer should be 24. Did you subtract accurately? Clear your abacus.	59 -12 47 -23 24 YES
15	There are three-digit numbers in this problem. Set 685 on your abacus. From that number subtract 372. Read me your answer. I agree. or The correct answer is 313. Clear your abacus.	685 -372 313
16	Try this subtraction problem. To begin set 586 on your abacus. From that number subtract 113, that is 1 hundred, 1 ten, and 3. The correct answer is 473. Was your answer correct? Clear your abacus.	586 -113 473 YES
17	This problem is similar to the last one. Begin by setting 657 on your abacus. From that number subtract 213. The correct answer is 444. Did you subtract correctly? You may clear your abacus.	657 -213 444 YES
18	Try this subtraction problem. Set 9,785 on your abacus. From that number subtract 4,251. Now subtract 2 thousands, 3 hundreds, 3 tens and 2. Read me your answer. I agree. or Your final answer should be 3,202. Clear your abacus.	9785 -4251 5534 -2332 3202

Criterion Check

I'm going to read five problems. I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{rrrrr} 55 & 56 & 5678 & 67 & 7596 \\ -31 & -31 & -1235 & -32 & -3142 \\ \hline 24 & 25 & 4443 & 35 & 4454 \\ & -12 & & -\ 2 & \\ \cline{2-2}\cline{4-4} & 13 & & 33 & \end{array}$$

---

19 This time let's work a problem that has several numbers in it. Set 556 on your abacus. From that number subtract 321. Now subtract 201.

Your final answer should be 34. Did you subtract accurately?

Clear your abacus.

$$\begin{array}{r} 556 \\ -321 \\ \hline 235 \\ -201 \\ \hline 34 \end{array}$$

---

20 Here's another problem similar to the last one. Set 695 on your abacus. From that number subtract 121. Finally, subtract 232. Read me your answer.

Good. or The correct answer is 342.

You may clear your abacus.

$$\begin{array}{r} 695 \\ -121 \\ \hline 574 \\ -232 \\ \hline 342 \end{array}$$

---

21 There are four-digit numbers in this problem. Set 6,565 on your abacus. From that number subtract 1 thousand, 1 hundred, 2 tens and 3.

Your final answer should be 5,442. Did you subtract accurately?

Clear your abacus.

$$\begin{array}{r} 6565 \\ -1123 \\ \hline 5442 \end{array}$$

YES

There are large numbers in this problem. Begin by setting 12,659 on your abacus. Now, subtract 10,313. Finally, subtract 2,222. Read me your answer.

$$\begin{array}{r} 12{,}659 \\ -10{,}313 \\ \hline 2{,}346 \\ -\ 2{,}222 \\ \hline 124 \end{array}$$

22 Good. or The correct answer is 124.

Clear your abacus.

Criterion Check

I'm going to read five problems. I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{rrrrr} 5559 & 556 & 15{,}678 & 6685 & 775 \\ -3214 & -131 & -11{,}235 & -3261 & -322 \\ \hline 2345 & 425 & 4{,}443 & 3424 & 453 \\ & -212 & & & -212 \\ \cline{2-2}\cline{5-5} & 213 & & & 241 \end{array}$$

LESSON THIRTY-ONE*

1	Let's begin this lesson by working a problem which has both addition and subtraction in it. Set 97 on your abacus. From that number subtract 3 tens and 3 or 33. Now subtract 23. This time add 94. Finally, subtract 31. Your final answer should be 104. Did you calculate correctly? Good. or Your final answer should be 104. Clear your abacus.	$\begin{array}{r} 97 \\ -33 \\ \hline 64 \\ -23 \\ \hline 41 \\ +94 \\ \hline 135 \\ -31 \\ \hline 104 \end{array}$ YES
2	Set 5 on your abacus. Now set 1 and clear 5. What number did you subtract when you set 1 and cleared 5? No comment. or You subtracted 4. Clear your abacus.	4
3	Set 6 on your abacus. Perhaps setting 1 and clearing 5 will help you show that 6 - 4 = 2. Try it. Did the rule work? No comment. or It should work. Clear your abacus.	YES
4	This time begin by setting 7 on your abacus. Will setting 1 and clearing 5 help you show that 7 - 4 = 3? Try it before you answer. No comment. or The rule should work. Clear your abacus.	YES

I have given you a sheet with three simple problems on it. Work each problem for yourself and then circle the problem or problems in which you set 1 and cleared 5 as a way of subtracting 4.

5 ANSWER STARRED

14 - 4 = 10
9 - 4 = 5
*8 - 4 = 4

You should have circled the third problem.

I'm going to read three phrases which describe abacus calculation. After I have read all three phrases set a 1, 2, or 3 to show which phrase you think explains a way of subtracting 4.

6 ANSWER STARRED

Set 2, clear 5
*Set 1, clear 5
Clear 1, set 5

To set 1 and clear 5 is a way of subtracting 4.

When you subtract 4 you should set 1 with your thumb. Show me the finger you should use to clear 5.

7 RIGHT INDEX

Good. or You should clear with your right index finger.

Here's a problem in which you'll need the rule set 1, clear 5 to subtract 4. Begin by setting 97 on your abacus. From that number subtract 64.

8 The difference is 33. Did you subtract correctly?

$$\begin{array}{r}97\\-64\\\hline 33\end{array}$$

Good. or Thirty-three is the correct answer.

YES

Clear your abacus.

---

9 Try this problem which will give you more than one chance to use the rule set 1, clear 5 as a way of subtracting 4. Set 68 on your abacus. Now subtract 4 tens and 4 or 44. Read me your answer.

$$\begin{array}{r} 68 \\ -44 \\ \hline 24 \end{array}$$

I agree. or The correct answer is 24.

Clear your abacus.

---

10 I have given you a sheet with three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used the rule set 1, clear 5 to subtract 4.

ANSWER STARRED

99 - 14 = 85
*25 - 14 = 11
44 - 14 = 30

You should have circled the second problem.

Criterion Check

I'm going to read five problems which you should be able to work. I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{rrrrr} 85 & 69 & 97 & 37 & 78 \\ -24 & -48 & -14 & -14 & -14 \\ \hline 61 & 21 & 83 & 23 & 64 \\ & & -42 & & -44 \\ \hline & & 41 & & 20 \end{array}$$

---

11 In this problem you'll need to use more than one rule of subtraction. Set 65 on your abacus. From that number subtract 4 tens and 2 or 42. Read me your answer.

$$\begin{array}{r} 65 \\ -42 \\ \hline 23 \end{array}$$

I agree. or Twenty-three is the correct answer.

Clear your abacus.

12

This time let's do a subtraction problem that has several numbers in it. Set 89 on your abacus. From that number subtract 4 tens and 3 or 43. Now subtract 23.

$$\begin{array}{r} 89 \\ -43 \\ \hline 46 \\ -23 \\ \hline 23 \end{array}$$

Your final answer should be 23. Did you subtract accurately?

Clear your abacus.

YES

13

There are three-digit numbers in this problem. Set 786 on your abacus. From that number subtract 453. Read me your answer.

$$\begin{array}{r} 786 \\ -453 \\ \hline 333 \end{array}$$

Fine. or The correct answer is 333.

Clear your abacus.

14

Try this problem. Set 688 on your abacus. Subtract 242. Now subtract 234. Read me your answer.

$$\begin{array}{r} 688 \\ -242 \\ \hline 446 \\ -234 \\ \hline 212 \end{array}$$

I agree. or I disagree your answer should be 212.

You may clear your abacus.

15

There are four-digit numbers in this problem. Set 5,558 on your abacus. From that number subtract 3,145.

$$\begin{array}{r} 5558 \\ -3145 \\ \hline 2413 \end{array}$$

The answer is 2,413. Did you subtract accurately?

Clear your abacus.

YES

16 Here's another problem with four-digit numbers in it. To begin set 6,987 on your abacus. From that number subtract 2 thousands, 4 hundreds, 3 tens and 3. Now subtract 2,432.

6987
-2433
4554
-2432
2122

Your final answer should be 2,122. Did you subtract correctly?

Clear your abacus.

YES

Criterion Check

I'm going to read five problems which you should be able to work. As usual, I'll ask you to read your results aloud and will repeat a problem if necessary.

58	59	875	8576	78
-42	-33	-431	-5342	-14
16	26	444	3234	64
- 2	-14			-34
14	12			30

I'm going to read four phrases that describe how to calculate on the abacus. Raise your hand each time you hear a true statement.

*Set 3, clear 5 is a way of subtracting 2.
That was a true statement.

17 *Set 1, clear 5 is a way of subtracting 4.
That was a true statement.

ANSWERS STARRED

*Set 2, clear 5 is a way of subtracting 3.
That was a true statement.

Set 1, clear 5 set 1 left is a way of subtracting 6.
That was a false statement.

18

Try this problem which has five-digit numbers in it. Begin by setting 15,856 on your abacus. From that number subtract 2 thousands, 4 hundreds, 1 ten and 3.

Your final answer should be 13,443. Did you subtract accurately?

Clear your abacus.

15,856
-2,413
13,443

YES

19

This problem has several numbers in it. To begin set 5,885 on your abacus. From that number subtract 2,211. Finally, subtract 3,342.

Your final answer should be 332. Did you subtract accurately?

Clear your abacus.

5885
-2211
3674
-3342
332

YES

Criterion Check

I'm going to read five problems which you should be able to work. As usual, I'll ask you to read your results aloud and will repeat a problem if necessary.

666	669	7777	976	788
-124	-344	-4263	-343	-432
542	325	3514	633	356
-321	-122	-2113	-222	-113
221	203	1401	411	243

LESSON THIRTY-TWO

1 In this lesson you'll learn a rule for subtracting 5. Before we begin something new let's try a problem in which you can use the subtraction rules which you've already learned. Begin by setting 75 on your abacus. From that number subtract 42. Now, add 24. Finally, subtract 13.

$$\begin{array}{r}75\\-42\\\hline 33\\+24\\\hline 57\\-13\\\hline 44\end{array}$$

Your final answer should be 44. Did you calculate accurately?

Clear your abacus. YES

---

2 Set 10 on your abacus. Now set 5 and clear 10. What number did you subtract when you set 5 and cleared 10?

5

No comment. or You subtracted 5.

Clear your abacus.

---

3 This time begin by setting 10 tens or 100 on your abacus. Now set 5 tens and clear 10 tens. What number did you subtract?

50

No comment. or You subtracted 50.

Clear your abacus.

---

4 I'm going to read three phrases. When I have finished set a 1, 2, or 3 on your abacus to show whether you think the first, second, or third phrase best explains a way of subtracting 5.

ANSWER STARRED

Set 4, clear 5
Set 1, clear 5
*Set 5, clear 1 left

To set 5 and clear 1 left is a way of subtracting 5.

5	Will the rule set 5, clear 1 left help you show that 20 - 5 = 15? Try it before you answer. No comment. or The rule should work. Clear your abacus.	YES
6	Set 45 on your abacus. Now subtract 5. Did you use the rule set 5, clear 1 left to subtract 5? No comment. or You could not have used the rule. Do not clear 40.	NO
7	Subtract 5 from 40. Did you use the rule set 5, clear 1 left to subtract 5? No comment. or You should have used the rule that time. Do not clear 35.	YES
8	I am going to count backwards from 35 by 5's. As I count, you should subtract 5. You must decide whether or not to apply the rule for subtracting 5.	OBV

Criterion Check

I'm going to read seven problems which you should be able to work. As usual, I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

91	134	430	43	194	226	106
-65	- 25	-155	-35	- 75	-156	- 55
26	109	275	8	119	70	51
	- 56			- 52	- 15	
	53			67	55	

9 I have given you a sheet which has three simple problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used the rule set 5, clear 1 left to subtract 5.

*11 - 5 = 6
9 - 5 = 4
*14 - 5 = 9

You should have circled the first and third problems.

ANSWERS STARRED

---

10 Here's another problem in which you can use the rule set 5, clear 1 left. Set 138 on your abacus. From that number subtract 5 tens and 5 or 55. Read me your answer.

Good. or The correct answer is 83.

Clear your abacus.

$$\begin{array}{r} 138 \\ -\ 55 \\ \hline 83 \end{array}$$

---

11 Set 42 on your abacus. Now, use the rule set 5, clear 1 left to subtract 5. The number 37 should be recorded on your abacus. Continue to subtract by 5's until either 2, 3, or 4 appears on your abacus.

Your final result should be 2. You may clear your abacus.

OBV

---

12 Try this problem. Begin by setting 319 on your abacus. From that number subtract 58. Now subtract 25. Be sure you look above and below the bar when you read your answer.

Good. or The correct answer is 236.

Clear your abacus.

$$\begin{array}{r} 319 \\ -\ 58 \\ \hline 261 \\ -\ 25 \\ \hline 236 \end{array}$$

---

13 You have used the rule set 5, clear 1 left to subtract in the ones column and in the tens column. In this problem you must apply that rule for subtracting 5 hundreds. Set 1,349 on your abacus. From that number subtract 526. Read me your answer.

$$\begin{array}{r}1349\\-526\\\hline 823\end{array}$$

Good. or The correct answer is 823.

Clear your abacus.

---

14 To work this problem you'll need to use more than one rule of subtraction. Set 316 on your abacus. From that number subtract 5 tens and 4 or 54. Read me your answer.

$$\begin{array}{r}316\\-54\\\hline 262\end{array}$$

I agree. or The correct answer is 262.

Clear your abacus.

---

15 Here's another problem in which you'll need more than one rule of subtraction. Set 652 on your abacus. From that number subtract 435.

$$\begin{array}{r}652\\-435\\\hline 217\end{array}$$

The difference is 217. Did you subtract accurately?

Clear your abacus. YES

---

16 This problem has several numbers in it. Begin by setting 651 on your abacus. From that number subtract 115. Now subtract 314.

$$\begin{array}{r}651\\-115\\\hline 536\\-314\\\hline 222\end{array}$$

Your final answer should be 222. Did you calculate accurately?

Clear your abacus. YES

Try this problem. Set 564 on your abacus. From that number subtract 4 tens and 2 or 42. Now subtract 315.

17 Your final answer should be 207. Did you calculate accurately?

Clear your abacus.

564
- 42
522
-315
207

YES

Here's another problem that is similar to the last one. Begin by setting 767 on your abacus. From that number subtract 226. Now subtract 135. Look above and below the bar when you read your answer. 18

Good. or The correct answer is 406.

Clear your abacus.

767
-226
541
-135
406

Criterion Check

I'm going to read eight problems which you should be able to work. Raise your hand if you would like me to repeat a problem. I'll ask you to read your answer aloud.

90	5933	905	560	71	9524	638	746
-35	-1652	- 54	-325	-15	-6115	-453	-154
55	4281	851	235	56	3409	185	592
-25		-531	-124	-23		- 23	-262
30		320	111	33		162	330

LESSON THIRTY-THREE

1	So far you've learned rules for subtracting 1, 2, 3, 4, and 5. What number do you think you'll learn to subtract in this lesson?	6
2	Before learning the new subtraction rule, let's work a problem in which you can practice all the subtraction rules and some of the addition rules you've already learned. Here's the problem. 60 - 5 - 21 - 32 + 55 - 13 Your final answer should be 44. Did you calculate accurately? Clear your abacus.	YES
3	Set 10 on your abacus to show that you are a boy or girl who has 10 cents to spend. If you spent 6 cents, how much money would you have left? Make your abacus say 4.	4 CENTS
4	To subtract 6 you set 4 lower beads and cleared 10. We can say that to set 4 and clear 1 left is a way of subtracting 6.	NONE
5	Here's a problem in which you can use the rule set 4, clear 1 left to subtract 6. Set 30 on your abacus. From that number subtract 1 ten and 6 or 16. Read me your answer. I agree. or Fourteen is the correct answer. Clear your abacus.	$\begin{array}{r}30\\-16\\\hline 14\end{array}$

Set 15 on your abacus. Will the rule set 4, clear 1 left help you show that 15 - 6 = 9? Try it before you answer.

6

No comment. or The rule should work.

YES

Clear your abacus.

Try this problem. You should look for a chance to use the rule set 4, clear 1 left. Set 95 on your abacus. From that number subtract 5 tens and 6 or 56.

$$\begin{array}{r} 95 \\ -56 \\ \hline 39 \end{array}$$

7

The difference is 39. Did you subtract correctly?

Clear your abacus.

YES

8

The rule for subtracting 6 says set 4, clear 1 left. You should set 4 with your thumb. Show me the finger you should use to clear 1 left.

RIGHT INDEX

Fine. or You should clear with your right index finger.

Because each column to the left on an abacus is worth ten times as much as the one before it, we can say set 4, clear 1 left instead of set 4, clear 10. That means that the rule can be applied in any column on the abacus.

9

As you work this problem, think about whether you are using the rule for subtracting 6 in the ones column, the tens column, the hundreds column, or the thousands column. Set 1,029 on your abacus. From that number subtract 615. The correct answer is 414. In what column did you apply the rule for subtracting 6?

$$\begin{array}{r} 1029 \\ -\ 615 \\ \hline 414 \end{array}$$

HUNDREDS

Good. or You should have applied the rule in the hundreds column.

10 This problem will give you more than one chance to use the rule for subtracting 6. Begin by setting 805 on your abacus. From that number subtract 666. Read me your answer.

$$\begin{array}{r} 805 \\ -666 \\ \hline 139 \end{array}$$

I agree. or The correct answer is 139.

Clear your abacus.

Criterion Check

I'm going to read six problems which you should be able to work. When you've finished, I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

90	755	152	208	152	151
-36	-566	- 62	- 61	- 62	- 61
54	189	90	147	90	90
		-26		-26	-66
		64		64	24

I have given you a sheet with three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used the rule set 4, clear 1 left to subtract 6.

11 ANSWERS STARRED

897 - 166 = 731
*400 - 166 = 234
*955 - 166 = 789

You should have circled the second and third problem.

12 This problem will give you a chance to use more than one rule of subtraction. Begin by setting 60 on your abacus. From that number subtract 3 tens and 6 or 36. Now subtract 5.

$$\begin{array}{r} 60 \\ -36 \\ \hline 24 \\ -\ 5 \\ \hline 19 \end{array}$$

Your final answer should be 19. Did you subtract accurately?

Clear your abacus.

YES

13 There are four-digit numbers in this problem. Set 6,259 on your abacus. From that number subtract 2,562. Look above and below the bar when you read your answer.

$$\begin{array}{r} 6259 \\ -2562 \\ \hline 3697 \end{array}$$

I agree. or The correct answer is 3,697.

Clear your abacus.

14 This problem has large numbers in it. Begin by setting 15,675 on your abacus. From that number subtract 6, 154.

$$\begin{array}{r} 15,675 \\ -\ 6,154 \\ \hline 9,521 \end{array}$$

Your final answer should be 9,521. Did you calculate accurately?

Clear your abacus.

15 This problem has three-digit numbers in it. Set 958 on your abacus. From that number subtract 362. Now subtract 334, that is 3 hundreds, 3 tens and 4. Read me your answer.

$$\begin{array}{r} 958 \\ -362 \\ \hline 596 \\ -334 \\ \hline 262 \end{array}$$

I agree. or The correct answer is 262.

Clear your abacus.

16	Let's do one more problem together. Think carefully so that you will work accurately. Begin by setting 7,180. From that number subtract 1 thousand, 5 hundreds, 6 tens and 6. Now subtract 1,414. Your final answer should be 4,200. Did you subtract accurately?	$\begin{array}{r} 7180 \\ -1566 \\ \hline 5614 \\ -1414 \\ \hline 4200 \end{array}$
	Clear your abacus.	YES

Criterion Check

I'm going to read nine problems. Raise your hand if you'd like me to repeat a problem. I'll ask you to read your answer aloud.

$$\begin{array}{r} 545 \\ -126 \\ \hline 419 \end{array} \quad \begin{array}{r} 7021 \\ -4615 \\ \hline 2406 \end{array} \quad \begin{array}{r} 10,978 \\ -6,532 \\ \hline 4,446 \end{array} \quad \begin{array}{r} 950 \\ -436 \\ \hline 514 \\ -414 \\ \hline 100 \end{array} \quad \begin{array}{r} 555 \\ -162 \\ \hline 393 \end{array} \quad \begin{array}{r} 650 \\ -516 \\ \hline 134 \end{array} \quad \begin{array}{r} 3052 \\ -1645 \\ \hline 1407 \end{array}$$

$$\begin{array}{r} 15,643 \\ -\ 6,325 \\ \hline 9,318 \end{array} \quad \begin{array}{r} 680 \\ -426 \\ \hline 254 \\ -135 \\ \hline 119 \end{array}$$

LESSON THIRTY-FOUR

1	Let's begin this lesson by working a problem in which you can use all you've learned about subtraction on the abacus. Here's the problem. 36 - 3 - 3 - 5 - 4 - 1 - 6 - 5 - 4 - 2 Your final answer should be 3. Did you calculate accurately? Clear your abacus.	YES
2	Set 10 on your abacus. Now set 3 and clear 10. What number did you subtract when you set 3 and cleared 10?	7
3	If to set 3 and clear 10 is a good way of subtracting 7, it should help you show that 20 - 17 = 3. Use your abacus to work that problem. Did the rule work? No comment. or It should have worked. Clear your abacus.	YES
4	Set 11 on your abacus. Perhaps the rule set 3, clear 1 left will help you show that 11 - 7 = 4. Try it. Did the rule work? No comment. or It should have worked. Clear your abacus.	YES
5	I have given you a sheet with three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems which you used the rule set 3, clear 1 left to subtract 7. 89 - 77 = 12 *15 - 7 = 8 98 - 76 = 22 You should have circled the second problem.	ANSWER STARRED

6 To say that the rule for subtracting 7 is set 3, clear 1 left rather than to say set 3, clear 10 is a way of saying that the rule may be used in any column on the abacus. As you work this problem think about whether you are using the rule for 7 in the ones column, the tens column, the hundreds column, or the thousands column. Set 102 on your abacus. Subtract 71. The correct answer is 31. In what column did you apply the rule for subtracting 7?

$$\begin{array}{r}102\\-\ 71\\\hline 31\end{array}$$

TENS

No comment. or You used the rule to subtract in the tens column.

---

7 In this problem you'll need to use the rule for subtracting 7 more than one time. Set 960 on your abacus. From that number subtract 1 hundred, 7 tens and 7 or 177. Read me your answer.

$$\begin{array}{r}960\\-177\\\hline 783\end{array}$$

I agree. or The correct answer is 783.

Clear your abacus.

---

8 Try this problem. Begin by setting 191 on your abacus. From that number subtract 3 tens and 7 or 37. Now subtract 72.

Your final answer should be 82. Did you subtract accurately?

$$\begin{array}{r}191\\-\ 37\\\hline 154\\-\ 72\\\hline 82\end{array}$$

Clear your abacus.

YES

---

9 I have given you a sheet with three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used the rule set 3, clear 1 left to subtract 7.

ANSWERS STARRED

* 45 - 27 = 18
*941 - 727 = 214
*109 - 78 = 31

You should have circled all three problems.

Criterion Check

I'm going to read five problems. I will ask you to read your answer aloud and will repeat a problem if necessary.

$$\begin{array}{rrrrr} 70 & 91 & 216 & 195 & 167 \\ -17 & -37 & -177 & -\ 27 & -\ 77 \\ \hline 53 & 54 & 39 & 168 & 90 \\ & & & -\ 77 & -77 \\ \cline{4-5} & & & 91 & 13 \end{array}$$

I am going to read three phrases which explain how to subtract on the abacus. What number can be subtracted with each rule? Set that number on your abacus.

10

Set 3, clear 1 left	7
Set 4, clear 1 left	6
Set 5, clear 1 left	5

No comment. or Restatement of rule indicating its application.

---

11 This problem will give you an opportunity to use more than one rule of subtraction. Set 155 on your abacus. From that number subtract 6 tens and 7 or 67. Read me your answer.

$$\begin{array}{r} 155 \\ -67 \\ \hline 88 \end{array}$$

Good. or Eighty-eight is the correct answer.

Clear your abacus.

---

12 Try this problem. Set 891 on your abacus. From that number subtract 4 hundreds, 6 tens and 7 or 467. Read me your answer.

$$\begin{array}{r} 891 \\ -467 \\ \hline 424 \end{array}$$

I agree. or The correct answer is 424.

Clear your abacus.

13 There are four-digit numbers in this problem. Begin by setting 6,966 on your abacus. From that number subtract 2,375.

$$\begin{array}{r}6966\\-2375\\\hline 4591\end{array}$$

Your final answer should be 4,591. Did you subtract accurately?

Clear your abacus.

YES

---

14 This problem also has four-digit numbers in it. Begin by setting 5,495. From that number subtract 1 thousand, 5 hundreds, 1 ten and 7. Look above and below the bar when you read your answer.

$$\begin{array}{r}5495\\-1517\\\hline 3978\end{array}$$

Fine. or The correct answer is 3,978.

Clear your abacus.

Criterion Check

I'm going to read five problems. I will ask you to read your answer aloud and will repeat a problem if necessary.

$$\begin{array}{rrrrr}181 & 105 & 646 & 8205 & 805\\ -\ 27 & -\ 64 & -257 & -2573 & -476\\ \cline{1-5} 154 & 41 & 389 & 5632 & 329\\ -\ 64 & -\ 17 & & & \\ \cline{1-2} 90 & 24 & & & \end{array}$$

---

15 This problem has large numbers in it. Begin by setting 16,598 on your abacus. From that number subtract 7,632.

$$\begin{array}{r}16{,}598\\7{,}632\\\hline 8{,}966\end{array}$$

Your final answer should be 8,966. Did you subtract accurately?

Clear your abacus.

YES

16 This problem has several numbers in it. To begin set 917 on your abacus. From that number subtract 6 hundreds, 7 tens and 3 or 673. Now subtract 52. Read me your answer.

I agree. or The correct answer is 192.

Clear your abacus.

$$\begin{array}{r} 917 \\ -673 \\ \hline 244 \\ -\ 52 \\ \hline 192 \end{array}$$

---

17 There are three- and four-digit numbers in this problem. Set 9,955 on your abacus. From that number subtract 874. Finally, subtract 651.

Your final answer should be 8,430. Did you get the correct answer?

Clear your abacus.

$$\begin{array}{r} 9955 \\ -\ 874 \\ \hline 9081 \\ -\ 651 \\ \hline 8430 \end{array}$$

YES

Criterion Check

I'm going to read five problems. I will ask you to read your answer aloud and will repeat a problem if necessary.

$$\begin{array}{rrrrr} 15{,}666 & 805 & 597 & 1558 & 508 \\ -\ 7{,}514 & -276 & -141 & -\ 622 & -162 \\ \hline 8{,}152 & 529 & 456 & 936 & 346 \\ -\ 6{,}532 & -218 & -267 & -\ 317 & -217 \\ \hline 1{,}620 & 311 & 189 & 619 & 129 \end{array}$$

LESSON THIRTY-FIVE

As usual let's begin this lesson by working a problem in which you can use what you've already learned about calculating on the abacus. Here's the problem.

96 - 22 - 15 - 12 - 23 - 14 - 6 + 82 - 7 - 45

1

Your final answer should be 34. Did you calculate accurately?

YES

Clear your abacus.

Set 10 on your abacus. You know that 10 - 8 = 2. Make your abacus say 2. What number did you subtract when you set 2 and cleared 10?

2

8

No comment. or You subtracted 8.

Clear your abacus.

This problem will give you a chance to subtract 8 by setting 2 and clearing 10. Set 40 on your abacus. From that number subtract 2 tens and 8 or 28.

3

$$\begin{array}{r}40\\-28\\\hline 12\end{array}$$

Twelve is the correct answer. Was your answer correct?

Clear your abacus.

YES

Look for a chance to subtract 8 by setting 2 and clearing 1 left as you work this problem. Set 900 on your abacus. From that number subtract 788. Read me your answer.

4

$$\begin{array}{r}900\\-788\\\hline 112\end{array}$$

I agree. or The correct answer is 112.

Clear your abacus.

I have given you a sheet with three very simple problems on it. Work each problem with your abacus and circle the problem or problems in which you used the rule set 2, clear 1 left as a way of subtracting 8.

5

ANSWERS STARRED

*11 - 8 = 3
9 - 8 = 1
*12 - 8 = 4

You should have circled the first and third problems.

This problem will give you several chances to use the rule for subtracting 8. Begin by setting 191 on your abacus. From that number subtract 28. Now, subtract 82.

6

191
- 28
163
- 82
81

Your final answer should be 81. Did you subtract accurately?

Clear your abacus.

YES

I have given you a sheet with three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used the rule set 2, clear 1 left to subtract 8.

7

ANSWERS STARRED

*25 - 8 = 17
*127 - 86 = 41
97 - 81 = 16

You should have circled the first two problems.

Try this problem. Set 857 on your abacus. From that number subtract 1 hundred, 8 tens and 8 or 188.

8

857
-188
669

The difference is 669. Did you calculate accurately?

Clear your abacus.

YES

9 This time begin by setting 962 on your abacus. From that number subtract 888. Read me your answer.

962
-888
74

I agree. or Seventy-four is the correct answer.

Clear your abacus.

Criterion Check

I'm going to read five problems which you should be able to work. When you've finished, I'll ask you to read your answer aloud. Raise your hand if you'd like me to repeat a problem.

95	31	80	476	40
-78	-18	-58	-288	- 8
17	13	22	188	32
		-18		-18
		4		14

---

10 I'm going to read three phrases which explain how to subtract on the abacus. To show that you know what number can be subtracted with each rule set that number on your abacus.

Set 3, clear 5	2
Set 1, clear 5	4
Set 4, clear 1 left	6

No comment. or Restatement of rule indicating its application.

---

11 This problem will give you an opportunity to use more than one rule of subtraction. Set 137 on your abacus. From that number subtract 5 tens and 8 or 58. Read me your answer.

137
- 58
79

Good. or Seventy-nine is the correct answer.

Clear your abacus.

12

Try this problem. Set 769 on your abacus. From that number subtract 388. Now subtract 247.

Your final answer should be 134. Did you subtract correctly?

Clear your abacus.

$$\begin{array}{r} 769 \\ -388 \\ \hline 381 \\ -247 \\ \hline 134 \end{array}$$

YES

13

There are four-digit numbers in this problem. Begin by setting 5,225 on your abacus. From that number subtract 1,856. Be sure you look above and below the bar when you read your answer.

I agree. or The correct answer is 3,369.

Clear your abacus.

$$\begin{array}{r} 5225 \\ -1856 \\ \hline 3369 \end{array}$$

Criterion Check

I'm going to read five problems which you should be able to work. I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{rrrrr} 117 & 92 & 560 & 100 & 70 \\ -\ 78 & -38 & -178 & -\ 68 & -35 \\ \hline 39 & 54 & 382 & 32 & 35 \\ & -42 & & & -28 \\ \cline{2-2}\cline{5-5} & 12 & & & 7 \end{array}$$

14

This problem has several numbers in it. Begin by setting 950 on your abacus. From that number subtract 145. Now, subtract 286. Your final answer should be 519.

Clear your abacus.

$$\begin{array}{r} 950 \\ -145 \\ \hline 805 \\ -286 \\ \hline 519 \end{array}$$

OBV

15 There are four- and five-digit numbers in this problem. Set 16,784 on your abacus. From that number subtract 7 thousands, 8 hundreds, 6 tens and 2.

16,784
- 7,862
8,922

Your final answer should be 8,922. Did you calculate accurately?

Clear your abacus.

YES

---

16 This problem is similar to the one you worked at the beginning of this lesson. It will give you an opportunity to use what you've learned about calculating on the abacus. Here's the problem. 85 - 21 - 5 - 38 - 8 + 143 - 70 - 2 - 24 - 6

Your final answer should be 54. Did you calculate accurately?

Clear your abacus.

YES

Criterion Check

I'm going to read five problems which you should be able to work. I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

11,757	617	6450	15,785	640
- 8,412	-281	-5276	- 8,461	-238
3,345	336	1174	7,324	402
	-157	- 832		-275
	179	342		127

LESSON THIRTY-SIX*

1	As usual let's begin this lesson by working a problem in which you can use what you've learned about adding and subtracting on the abacus. 191 - 15 - 4 - 18 - 23 - 7 + 2 - 23 - 63 + 5 - 1 Your final answer should be 44. Did you calculate accurately? Clear your abacus.	YES
2	You've learned how to subtract 1, 2, 3, 4, 5, 6, 7, and 8. It should be easy to guess that in this lesson you'll practice a way of subtracting 9.	NONE
3	To subtract 6 you set 4 and clear 1 left. To subtract 7 you set 3 and clear 1 left. To subtract 8 you set 2 and clear 1 left. To subtract 9 you would need to set what number before clearing 1 left? Use your fingers to show that you know the answer to that question.	1
4	It is possible to subtract 9 by setting 1 and clearing 1 left. Try it in this problem. Set 70 on your abacus and then subtract 9 from it. The number 61 should be recorded on your abacus. Do not clear your abacus.	OBV
5	Subtract 9 from 61. The number 52 should be recorded on your abacus. Clear your abacus so you'll be ready for another problem.	OBV

This problem will give you two opportunities to use the rule set 1, clear 1 left for subtracting 9. Set 123 on your abacus. From that number subtract 9 tens and 9 or 99. Read me your answer.

$$\begin{array}{r} 123 \\ -\ 99 \\ \hline 24 \end{array}$$

6

I agree. or Twenty-four is the correct answer.

Clear your abacus.

I have given you a sheet with three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used the rule set 1, clear 1 left to subtract 9.

ANSWERS STARRED

7 *809 - 798 = 11
*486 - 129 = 357
*1272 - 991 = 281

You should have circled all three problems.

Criterion Check

I'm going to read four problems. Raise your hand if you'd like me to repeat the problem. I'll ask you to read your answer aloud.

$$\begin{array}{r} 40 \\ -29 \\ \hline 11 \\ -\ 9 \\ \hline 2 \end{array} \qquad \begin{array}{r} 82 \\ -19 \\ \hline 63 \\ -59 \\ \hline 4 \end{array} \qquad \begin{array}{r} 1678 \\ -\ 999 \\ \hline 679 \end{array} \qquad \begin{array}{r} 96 \\ -29 \\ \hline 67 \\ -\ 9 \\ \hline 58 \end{array}$$

In this problem you'll need to use the rule for subtracting 9 and one other rule of subtraction. Set 161 on your abacus. From that number subtract 9 tens and 8 or 98.

8 Read me your answer.

$$\begin{array}{r} 161 \\ -\ 98 \\ \hline 63 \end{array}$$

I agree. or The correct answer is 63.

You may clear your abacus.

9	This time let's subtract several numbers. Begin by setting 77 on your abacus. From that number subtract 1 ten and 9 or 19. Now subtract 17. Your final answer should be 41. Was your answer correct? Clear your abacus.	77 -19 58 -17 41 YES
10	There are three-digit numbers in this problem. Set 678 on your abacus. From that number subtract 219. Look above and below the bar when you read your answer. Good. or The correct answer is 459. Clear your abacus.	678 -219 459
11	Try this problem. Set 932 on your abacus. From that number subtract 159. Now, subtract 392. Your final answer should be 381. Did you subtract accurately? Clear your abacus.	932 -159 773 -392 381 YES
12	There are four-digit numbers in this problem. Begin by setting 7,596 on your abacus. From that number subtract 4,619. Look above and below the bar when you read your answer. Fine. or The correct answer is 2,977. Clear your abacus.	7596 -4619 2977

Criterion Check

I'm going to read six problems. I'll ask you to read your answer aloud and will repeat a problem if necessary.

93	65	456	7501	100	707
-29	-38	-269	-4179	- 85	-493
64	27	187	3322	15	214
-25	- 9			- 9	
39	18			6	

13

I'm going to read four phrases which explain how to subtract on the abacus. To show that you know what number can be subtracted with each rule set that number on your abacus.

Set 1, clear 5	4
Set 3, clear 1 left	7
Set 4, clear 5	1
Set 4, clear 1 left	6

No comment. or Restatement of rule indicating its application.

14

Try this problem which has several numbers in it. Begin by setting 705 on your abacus. From that number subtract 473. Now, subtract 198.

705
-473
232
-198
34

Your final answer should be 34. Did you subtract accurately?

Clear your abacus.

YES

15

Here's another problem similar to the last one. To begin set 928 on your abacus. Now, subtract 3 hundreds, 5 tens and 9. Finally, subtract 183. Read me your answer.

928
-359
569
-183
386

Good. or The correct answer is 386.

Clear your abacus

---

16 This problem is similar to the last one. Begin by setting 767 on your abacus. From that number subtract 492. Now subtract 183.

Your final answer should be 92. Did you subtract accurately?

Clear your abacus.

767
-492
275
-183
92

YES

---

17 This problem has four- and five-digit numbers in it. Begin by setting 12,756 on your abacus. From that number subtract 9,162. Be sure you look above and below the bar when you read your answer.

12,756
- 9,162
3,594

I agree. or The correct answer is 3,594.

Clear your abacus.

Criterion Check

I'm going to read five problems which you should be able to work. When you've finished, I'll ask you to read your answer aloud. I will repeat a problem if necessary.

655	1111	10,779	1258	13,538
-132	- 789	- 6,943	- 589	- 9,624
523	322	3,836	669	3,914
-109	- 152		- 572	
414	170		97	

LESSON THIRTY-SEVEN*

Before beginning something new, let's try a problem that will give you an opportunity to use everything you've learned about subtraction. 159 - 20 - 3 - 8 - 93 - 7 - 14 - 4 - 6 + 50 - 14 + 5 - 3 - 5.

1

Your final answer should be 37. Did you calculate accurately?

Clear your abacus. YES

Set 10 your your abacus. Use your thumb and index finger to set 9 and clear 10. First your abacus said 10. Then you made it say 9. What number did you subtract?

2 1

We may summarize what you did by saying that to set 9 and clear 10 is a way of subtracting 1.

3 I'm going to read a sentence but will leave out the last word in that sentence. I want you to tell me what word should be used to complete the sentence. Because each column to the left on an abacus is worth ten times as much as the one before it, instead of saying that the rule for subtracting 1 is set 9, clear 10 we say, "Set 9, clear 1 __________." LEFT

4 Since the rule for subtracting 1 says clear 1 left, it can be applied in any column on the abacus. Try it in this problem. Set 100 on your abacus. From that number subtract 1 ten. OBV

Clear 90 from your abacus.

5

This problem has three- and four-digit numbers in it and will give you a chance to use the rule set 9, clear 1 left. Begin by setting 4,098 on your abacus. From that number subtract 3,121.

$$\begin{array}{r} 4098 \\ -3121 \\ \hline 977 \end{array}$$

Your final answer should be 977. Did you subtract accurately?

Clear your abacus.

YES

6

I have given you a sheet with three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used the rule set 9, clear 1 left to subtract 1.

ANSWERS STARRED

87 - 41 = 46
*208 - 111 = 97
*806 - 216 = 590

You should have circled the last two problems.

7

There is more than one way to subtract 1 on an abacus. I'm going to read four phrases which explain how to calculate on the abacus. Raise your hand each time you hear a phrase which explains a way of subtracting 1.

ANSWERS STARRED

*Set 4, clear 5
I agree. or To set 4 and clear 5 is a way of subtracting 1.

Clear 4, set 5
No comment. or That is a way of adding 1.

*Set 9, clear 1 left
I agree. or That is a way of subtracting 1.

Set 3, clear 1 left
No comment. or That is a way of subtracting 7.

---

8 To work this problem you'll need to use more than one rule for subtracting 1. Begin by setting 508 on your abacus. From that number subtract 111. Look above and below the bar when you read your answer.

$$\begin{array}{r} 508 \\ -111 \\ \hline 397 \end{array}$$

Good. or The correct answer is 397.

Clear your abacus.

Criterion Check

I'm going to read six problems. I'll ask you to read your answer aloud. Raise your hand if you'd like me to repeat a problem.

$$\begin{array}{rrrrrr} 40 & 209 & 4506 & 180 & 102 & 670 \\ -21 & -\ 17 & -2115 & -\ 71 & -\ 12 & -\ 61 \\ \hline 19 & 192 & 2391 & 109 & 90 & 609 \\ & & & -\ 11 & & -\ 12 \\ \cline{4-4}\cline{6-6} & & & 98 & & 597 \end{array}$$

---

9 To work this problem you'll need to use more than one rule of subtraction. Begin by setting 150 on your abacus. From that number subtract 2 tens and 1 or 21. Read me your answer.

$$\begin{array}{r} 150 \\ -\ 21 \\ \hline 129 \end{array}$$

I agree. or The correct answer is 129.

Clear your abacus.

---

10 This problem will also give you an opportunity to use more than one rule of subtraction. Begin by setting 157 on your abacus. From that number subtract 6 tens and 7 or 67. Now subtract 31.

$$\begin{array}{r} 157 \\ -\ 67 \\ \hline 90 \\ -\ 31 \\ \hline 59 \end{array}$$

Your final answer should be 59. Did you subtract accurately?

Clear your abacus. YES

11 There are four-digit numbers in this problem. Set 8,260 on your abacus. From that number subtract 4 thousands, 5 hundreds, 8 tens and 1. Look above and below the bar when you read your answer.

$$\begin{array}{r} 8260 \\ -4581 \\ \hline 3679 \end{array}$$

I agree. or The correct answer is 3,679.

Clear your abacus.

---

12 This problem has several numbers in it. Begin by setting 1,056 on your abacus. From that number subtract 111. Now subtract 729.

$$\begin{array}{r} 1056 \\ -\ 111 \\ \hline 945 \\ -\ 729 \\ \hline 216 \end{array}$$

Your final answer should be 216. Did you calculate accurately?

Clear your abacus.

YES

---

13 There are four- and five-digit numbers in this problem. Begin by setting 11,730 on your abacus. From that number subtract 7,891. Be sure you look above and below the bar when you read your answer.

$$\begin{array}{r} 11{,}730 \\ -\ 7{,}891 \\ \hline 3{,}839 \end{array}$$

I agree. or The correct answer is 3,839.

Clear your abacus.

Criterion Check

I'm going to read nine problems which you should be able to work. I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{rrrrrrrrr} 178 & 226 & 10{,}558 & 900 & 52 & 126 & 10{,}196 & 930 & 1503 \\ -\ 88 & -\ 17 & -\ 1{,}983 & -161 & -12 & -\ 23 & -\ 1{,}948 & -221 & -\ 615 \\ \hline 90 & 209 & 8{,}575 & 739 & 40 & 103 & 8{,}248 & 709 & 888 \\ -\ 21 & -\ 13 & & -556 & -21 & -\ 15 & & -495 & -\ 333 \\ \cline{1-2}\cline{4-6}\cline{8-9} 69 & 196 & & 183 & 19 & 88 & & 214 & 555 \end{array}$$

LESSON THIRTY-EIGHT

1	Perhaps the best way to explain what we'll do in this lesson is to begin by working an addition problem. Set 58 on your abacus. To that number add 4 tens and 7 or 47. Your final answer should be 105. You may clear your abacus.	$\begin{array}{r} 58 \\ +47 \\ \hline 105 \end{array}$ OBV
2	To work the last problem you used a rule for adding 1 when it was necessary to set 1 left. We can say that you used one addition rule on top of another. In this lesson you'll learn to use one subtraction rule on top of another.	NONE
3	Set 50 on your abacus so that we can talk about how to subtract 6 from it. You know that the rule for subtracting 6 is set 4, clear 1 left. Set 4 as if you want to subtract 6.	OBV
4	Now move to the tens column. Is there a lower bead in the tens column which you can clear?	NO
5	Because there is not a lower bead you can clear from the bar, you will need to use a rule for subtracting 1 to help you. Do it. The number 44 should be recorded on your abacus. Clear your abacus.	OBV
6	What rule for subtracting 1 did you use to clear 1 left? No comment. To set 4 and clear 5 is a way of subtracting 1.	SET 4, CLEAR 5

7 This problem is similar to the last one. Set 62 on your abacus. From that number subtract 18. When it is time to clear 1 left, you should use the rule set 4, clear 5 to help you. Set 62 on your abacus. From that number subtract 1 ten and 8 or 18.

$$\begin{array}{r} 62 \\ -18 \\ \hline 44 \end{array}$$

Your final answer should be 44. Did you subtract accurately?

Clear your abacus.

YES

8 Try this problem. To begin set 738 on your abacus. Look for a chance to use the new way of clearing 1 left as you subtract 295 from that number. Your final answer should be 443.

$$\begin{array}{r} 738 \\ -295 \\ \hline 443 \end{array}$$

Clear your abacus.

9 I have given you a sheet with three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used a rule for subtracting 1 when it was time to clear 1 left.

57 - 25 = 32
*80 - 37 = 43
*90 - 45 = 45

ANSWERS STARRED

You should have circled the second and third problems.

10 Set 90 on your abacus. Subtract 1 from that number so that your abacus will say 89. What rule did you use to subtract 1 in that case?

No comment. or You subtracted 1 by setting 9 and clearing 1 left.

SET 9, CLEAR 1 LEFT

Clear your abacus.

In some cases you may need the rule set 9, clear 1 left to help you when it is time to clear 1 left. Let's try it.

11 Set 100 on your abacus. To show that 100 - 8 = 92 you would need to set 2 and clear 1 left. Set 2. When you are ready to clear 1 left use the rule set 9, clear 1 left to help you. OBV

Clear 92.

12 Here's another problem in which you will need to use a rule for 1 when it is time to clear 1 left. Set 120 on your abacus. From that number subtract 2 tens and 5 or 25.

120
- 25
95

Your final answer should be 95. Did you subtract accurately?

Clear your abacus. YES

13 Try this problem which is similar to the last one. Begin by setting 132 on your abacus. From that number subtract 39. Read me your answer.

132
- 39
93

I agree. or Your answer should be 93.

Clear your abacus.

14 You may use either the rule set 4, clear 5 or the rule set 9, clear 1 left to help you clear 1 left.

I have given you a sheet with three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used a rule for subtracting 1 when it was time to clear 1 left. ANSWERS STARRED

*60 - 19 = 41
*135 - 36 = 99
55 - 23 = 32

You should have circled the first and second problems.

15 This problem has three-digit numbers in it. Set 769 on your abacus. From that number subtract 274. Look above and below the bar when you read your answer.

$$\begin{array}{r} 769 \\ -274 \\ \hline 495 \end{array}$$

I agree. or The correct answer is 495.

Clear your abacus.

Criterion Check

I'm going to read eight problems. Raise your hand if you'd like me to repeat a problem. I'll ask you to read your answer aloud.

$$\begin{array}{rrrrrrrr} 94 & 460 & 126 & 775 & 92 & 786 & 181 & 975 \\ -45 & -267 & -\ 28 & -129 & -49 & -588 & -\ 87 & -626 \\ \hline 49 & 193 & 98 & 646 & 43 & 198 & 94 & 349 \end{array}$$

---

16 I am going to read you a long problem which will give you a chance to use all you know about subtracting on the abacus. I will read slowly so that you will have plenty of time to work accurately. Here's the problem. 101 - 9 - 22 - 35 - 12 - 5 - 4 - 3 - 7 - 4 + 51 - 8 - 13 - 16 + 21 - 1

Your final answer should be 34. Did you calculate accurately?

Clear your abacus. YES

---

17 Try this problem which has four-digit numbers in it. Set 8,168 on your abacus. From that number subtract 5 thousands, 1 hundred, 7 tens and 3. Be sure you look above and below the bar when you read your answer.

$$\begin{array}{r} 8168 \\ -5173 \\ \hline 2995 \end{array}$$

I agree. or The correct answer is 2,995.

Clear your abacus.

This problem is similar to the last one. Begin by setting 6,269 on your abacus. From that number subtract 1,543.

18 Your final answer should be 4,726. Did you subtract accurately?

6269
-1543
4726

Clear your abacus.

To begin this problem set 10,515 on your abacus. From that number subtract 892.

10,515
- 892
9,623

19 Your final answer should be 9,623. Did you subtract accurately?

Clear your abacus.

YES

Criterion Check

I'm going to read seven problems. I will ask you to read your answer aloud and will repeat a problem if necessary.

71	100	10,652	93	243	12,596	751
-18	- 5	- 711	-49	-149	- 2,873	-267
53	95	9,941	44	94	9,723	484
-42	- 62		-25			
11	33		19			

LESSON THIRTY-NINE

1 As usual let's begin this lesson by working a problem in which you can use what you've learned earlier. 78 - 14 - 23 - 7 -15 - 4 - 3 - 2 - 1 + 21 - 9 - 8 + 2 - 6 - 4 - 1

Your final answer should be 4. Did you calculate accurately?

Clear your abacus. YES

2 Set 10 on your abacus. If you wanted to subtract 2 from 10 could you do it by clearing 2 lower beads from the bar? NO

There is another way of subtracting 2 which we will learn in this lesson.

3 The number 10 should be recorded on your abacus. Now make it say 8. OBV

4 I'm going to read three phrases which explain how to move beads on the abacus. Raise your hand when you hear a phrase which explains what you did to subtract 2.

Set 8
Set 5
*Set 8 and cleared 10

ANSWER STARRED

We can summarize what you did to subtract 2 by saying that you set 8 and cleared 10. To show that the rule will apply in any column on the abacus, we say that you set 8 and cleared 1 left.

5 Here's a problem in which you can use the rule set 8, clear 1 left to subtract 2. Begin by setting 30 on your abacus. Subtract 2 from that number. Your answer should be 28. OBV

Clear your abacus.

6 This time set 21 on your abacus. From that number subtract 1 ten and 2. Try the rule set 8, clear 1 left when it is time to subtract 2. If the rule is a good one in that case, the number 9 should be recorded on your abacus. Is the rule a good one?

$$\begin{array}{r}21\\-12\\\hline 9\end{array}$$

No comment. or Yes, the rule is a good one. YES

Clear your abacus.

---

7 Think carefully as you work this problem. You'll need to use a rule for subtracting 2 and a rule for subtracting 1 when it is time to clear 1 left. Set 81 on your abacus. From that number subtract 22. Read me your answer.

$$\begin{array}{r}81\\-22\\\hline 59\end{array}$$

I agree. or The correct answer is 59.

Clear your abacus.

---

8 I'm going to read three phrases which explain how to move beads on the abacus. Raise your hand each time you hear a phrase which explains a way of subtracting 2.

Set 8, clear 1 left
I agree. or That is a way of subtracting 2.

Set 2, clear 1 left
No comment. or That is a way of subtracting 8.

Set 3, clear 5
I agree. or That is a way of subtracting 2.

---

9 To work this problem you'll need to use both rules for subtracting 2. Set 61 on your abacus. From that number subtract 2 tens and 2 or 22. Look above and below the bar when you read your answer.

$$\begin{array}{r}61\\-22\\\hline 39\end{array}$$

I agree. or The correct answer is 39.

Clear your abacus.

Criterion Check

I'm going to read eight problems. I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{rrrrrrrr}
509 & 91 & 71 & 211 & 916 & 80 & 81 & 460\\
-221 & -22 & -22 & -\ 12 & -322 & -12 & -32 & -162\\
\hline
288 & 69 & 49 & 199 & 594 & 68 & 49 & 298\\
 & -25 & & & & -23 & & \\
\cline{2-2}\cline{6-6}
 & 44 & & & & 45 & &
\end{array}$$

I'm going to read several phrases which explain how to subtract on the abacus. To show that you know what number can be subtracted with each rule set that number on your abacus.

10

Set 8, clear 1 left	2
Set 4, clear 5	1
Set 3, clear 1 left	7
Set 9, clear 1 left	1
Set 1, clear 1 left	9

I agree. or Restatement which indicates application of rule.

---

11 You'll need to use more than one rule of subtraction to work this problem. Set 861 on your abacus. From that number subtract 672. Read me your answer.

$$\begin{array}{r}861\\-672\\\hline 189\end{array}$$

I agree. or The correct answer is 189.

Clear your abacus.

There are three-digit numbers in this problem. You will need to use a rule for subtracting 1 when it is time to clear 1 left. Set 181 on your abacus. From that number subtract 82.

$$\begin{array}{r}181\\-\ 82\\\hline 99\end{array}$$

12 Your final answer should be 99. Did you subtract accurately?

YES

Clear your abacus.

13	Think carefully as you work this problem. Begin by setting 903 on your abacus. From that number subtract 412. Now subtract 262. Your final answer should be 229. Did you calculate accurately? Clear your abacus.	903 -412 491 -262 229 YES
14	There are three-digit numbers in this problem. Begin by setting 705 on your abacus. From that number subtract 186. Look above and below the bar when you read your answer. I agree. or The correct answer is 519. Clear your abacus.	705 -186 519

Criterion Check

I'm going to read seven problems. I'll repeat a problem if necessary and will ask you to read your results aloud.

100	201	607	7483	156	215	661
- 26	- 72	-324	-4365	- 48	- 86	-272
74	129	283	3118	108	129	389
- 32	- 88		-2226	- 28	- 18	
42	41		892	80	111	
- 2				- 1	- 2	
40				79	109	
- 1						
39						

LESSON FORTY

1 In this lesson you'll learn a way of subtracting 3 on the abacus. Before we begin something new, however, let's work a problem which will give you a chance to practice what you've already learned. 91 - 2 - 42 - 23 - 5 - 4 - 2 - 3 - 1 - 4 - 1 + 41 - 6 - 1 - 9 - 2 - 8 - 4 - 7 - 3 Your final answer should be 5. OBV

Clear your abacus.

2 Set 10 on your abacus. Now make your abacus say 7. What number did your subtract? 3

3 I'm going to read three phrases which describe a way of moving beads on the abacus. When I have finished, set a 1, 2, or 3 on your abacus to show whether you think the first, second, or third phrase best explains a way of subtracting 3. ANSWER STARRED

Set 2, set 5
*Set 7, clear 1 left
Set 8, clear 1 left

I agree. or To set 7 and clear 1 left is a way of subtracting 3.

4 Let's work a problem in which you can subtract 3 by setting 7 and clearing 1 left. Set 80 on your abacus. From that number subtract 1 ten and 3 or 13.

$$\begin{array}{r}80\\-13\\\hline 67\end{array}$$

The correct answer is 67. Did you subtract accurately?

Clear your abacus. YES

Here's another problem in which you can use the rule set 7, clear 1 left to subtract 3. Set 91 on your abacus. Subtract 73 from that number. Look above and below the bar
5 when you read your answer.

$$\begin{array}{r}91\\-73\\\hline 18\end{array}$$

Good. or The correct answer is 18.

Clear your abacus.

I have given you a sheet with three problems on it. The correct answers have been given. Work each problem for yourself and tnen circle the problem or problems in which you used the rule set 7, clear 1 left to subtract 3.
6

ANSWERS STARRED

*124 - 32 = 92
* 21 - 13 = 8
99 - 35 = 64

You should have circled the first two problems.

One way of subtracting 3 is to set 7 and clear 1 left. What is the other rule for subtracting 3?
7

SET 2, CLEAR 5

Fine. or To set 2 and clear 5 is a way of subtracting 3.

To work this problem you'll need to use both ways of subtracting 3. Set 62 on your abacus. From that number subtract 33. Read me your answer.
8

$$\begin{array}{r}62\\-33\\\hline 29\end{array}$$

Good. or Twenty-nine is the correct answer.

Clear your abacus.

Try this problem which has several numbers in it. Set 181 on your abacus. From that number subtract 13. Now subtract 35.

9 Your final answer should be 133. Did you subtract accurately?

Clear your abacus.

$$\begin{array}{r}181\\-\ 13\\\hline 168\\-\ 35\\\hline 133\end{array}$$

To work this problem you'll need to use a rule for subtracting 1 to help you when it is time to clear 1 left. Set 931 on your abacus. From that number subtract 3 hundreds, 3 tens and three or 333. Look above and below the bar when you read your answer. 10

$$\begin{array}{r}931\\-333\\\hline 598\end{array}$$

Good. or The correct answer is 598.

Clear your abacus.

Criterion Check

I'm going to read seven problems. I'll ask you to read your answer aloud and will repeat a problem if necessary.

40	626	108	671	81	118	580
-13	-533	- 35	-373	-23	- 32	-383
27	93	73	298	58	86	197
-13		- 31		-36	- 33	
14		42		22	53	

This problem will give you an opportunity to use several rules of subtraction. Begin by setting 60 on your abacus. From that number subtract 4 tens and 3 or 43. Now 11 subtract 8.

Your final answer should be 9. Did you subtract accurately?

Clear your abacus.

$$\begin{array}{r}60\\-43\\\hline 17\\-\ 8\\\hline 9\end{array}$$

YES

12 There are three-digit numbers in this problem. Set 529 on your abacus. From that number subtract 233. Look above and below the bar when you read your answer.

$$\begin{array}{r} 529 \\ -233 \\ \hline 296 \end{array}$$

Good. or The correct answer is 296.

Clear your abacus.

13 There are five-digit numbers in this problem. To begin set 10,000 on your abacus. From that number subtract 3 thousands, 6 hundreds, 2 tens and 8. Read me your answer.

$$\begin{array}{r} 10{,}000 \\ -\ 3{,}628 \\ \hline 6{,}372 \end{array}$$

I agree. or The correct answer is 6,372.

Clear your abacus.

Criterion Check

I'm going to read eight problems. I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

$$\begin{array}{rrrrrrrr} 107 & 9506 & 611 & 1104 & 702 & 4171 & 1061 & 700 \\ -\ 39 & -3638 & -173 & -\ 314 & -138 & -2392 & -\ 263 & -112 \\ \hline 68 & 5868 & 438 & 790 & 564 & 1779 & 798 & 588 \\ -\ \ 9 & & & -\ 553 & -312 & & & -133 \\ \cline{1-1}\cline{4-5}\cline{8-8} 59 & & & 237 & 252 & & & 455 \end{array}$$

LESSON FORTY-ONE*

As usual let's begin this lesson by reviewing what you've learned in other lessons. This time I will read three phrases which explain how to subtract on the abacus. To show that you know what number can be subtracted with each rule set that number on your abacus.

1

Set 7, clear 1 left	3
Set 8, clear 1 left	2
Set 9, clear 1 left	1

Good. or Restatement of rule indicating its application.

2 Now let's work a problem which will give you a chance to use what you know about subtracting on the abacus. I'll read slowly so that you will have plenty of time to work accurately. 100 - 23 - 4 - 5 - 9 - 12 - 3 - 4 - 17 - 3 -16 + 27 - 8 - 3 - 1 - 4 - 2 Your final answer should be 13. OBV

Clear your abacus.

3 Set 10 on your abacus. Now, make your abacus say 6. You know that the difference between 10 and 6 is 4 and so we can say that you have subtracted four. OBV

4 I'm going to read three phrases which explain how to move beads on the abacus. When I have finished set 1, 2, or 3 on your abacus to show whether you think the first, second, or third phrase best explains what you did to subtract 4.

Set 1, set 5
*Set 6, clear 1 left
Set 4, clear 1 left

ANSWER STARRED

To set 6 and clear 1 left is a way of subtracting 4.

5 To work this problem you will need to subtract 4 by setting 6 and clearing 1 left. Set 23 on your abacus. Now, subtract 14. The number 9 should be recorded on your abacus.

$$\begin{array}{r} 23 \\ -14 \\ \hline 9 \end{array}$$

Clear your abacus.

OBV

---

6 Here's another problem in which you can use the rule set 6, clear 1 left to subtract 4. Set 91 on your abacus. From that number subtract 6 tens and 4 or 64. Look above and below the bar when you read your answer.

$$\begin{array}{r} 91 \\ -64 \\ \hline 27 \end{array}$$

Good. or The correct answer is 27.

Clear your abacus.

---

7 This problem will give you more than one chance to use the rule set 6, clear 1 left to subtract 4. Begin by setting 109 on your abacus. From that number subtract 4 tens and 7 or 47. Now subtract 54.

$$\begin{array}{r} 109 \\ -\ 47 \\ \hline 62 \\ -\ 54 \\ \hline 8 \end{array}$$

Your final answer should be 8. Did you subtract accurately?

Clear your abacus.

YES

---

8 I have given you a sheet with three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used the rule set 6, clear 1 left to subtract 4.

ANSWERS
STARRED

*190 - 74 = 116
*126 - 45 = 81
94 - 42 = 52

You should have circled the first two problems.

To set 6 and clear 1 left is one way of subtracting 4. I'm going to read three phrases. When I have finished, set a 1, 2, or 3 on your abacus to show which phrase explains another way of subtracting 4.

ANSWER STARRED

9 Set 4
Clear 1, set 5
*Set 1, clear 5.

Good. or To set 1 and clear 5 is a way of subtracting 4.

In this problem you'll need to use both ways of subtracting 4. Set 153 on your abacus. From that number subtract 4 tens and 4 or 44. Look above and below the bar when you read
10 your answer.

153
- 44
109

I agree. or The correct answer is 109.

Clear your abacus.

Criterion Check

I'm going to read six problems which you should be able to work. Raise your hand if you'd like me to repeat a problem. I'll ask you to read your answer aloud.

136	62	113	342	539	226
- 44	-14	- 42	-144	-442	-145
92	48	71	198	97	81
		- 41			- 40
		30			41

There are three-digit numbers in this problem. Set 809 on your abacus. From that number subtract 444. Read me your answer.

809
-444
365

11

I agree. or The correct answer is 365.

Clear your abacus.

I'm going to read four statements. If what I say is true you should raise your hand. If what I say is not true sit quietly.

The difference between 9 and 10 is 1 and so you can subtract 1 by setting 9 and clearing 1 left. TRUE

The difference between 8 and 10 is 2 and so you can subtract 2 by setting 8 and clearing 1 left. TRUE

12 The difference between 7 and 10 is 3 and so you can subtract 3 by setting 7 and clearing 1 left. TRUE

The difference between 6 and 10 is 4 and so you can subtract 4 by setting 6 and clearing 1 left. TRUE

All of the statements were true. It is possible to subtract on the abacus by thinking about the difference between 10 and the number you want to subtract. When you add the difference and clear 10 you will have subtracted correctly.

Try this problem which has several numbers in it. Begin by setting 131 on your abacus. From that number subtract 43. Now subtract 14.

13

$$\begin{array}{r}131\\-\ 43\\\hline 88\\-\ 14\\\hline 74\end{array}$$

Your final answer should be 74. Did you subtract accurately?

Clear your abacus. YES

14 Here's another problem similar to the last one. Set 71 on your abacus. From that number subtract 14. Now subtract 48. Your final answer should be 9.

$$\begin{array}{r}71\\-14\\\hline 57\\-48\\\hline 9\end{array}$$

Clear your abacus. OBV

15 This problem has three- and four-digit numbers in it. Set 1,153 on your abacus. From that number subtract 254. Read me your answer.

Good. or The correct answer is 899.

Clear your abacus.

$$\begin{array}{r}1153\\ -\ 254\\ \hline 899\end{array}$$

---

16 Let's try a problem that has four-digit numbers in it. Set 6,001 on your abacus. From that number subtract 5,432. Read me your answer.

Fine. or The correct answer is 569.

Clear your abacus.

$$\begin{array}{r}6001\\ -5432\\ \hline 569\end{array}$$

---

17 This problem has four- and five-digit numbers in it. Begin by setting 10,730 on your abacus. From that number subtract 7,941.

Your answer should be 2,789. Did you subtract accurately?

Clear your abacus.

$$\begin{array}{r}10{,}730\\ -\ 7{,}941\\ \hline 2{,}789\end{array}$$

YES

Criterion Check

I'm going to read nine problems which you should be able to work. I'll ask you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

141	121	750	9531	12,102	106	730	4906	11,220
- 24	- 42	-591	-6594	- 8,274	- 45	-142	-2438	- 5,481
117	79	159	2937	3,828	61	588	2468	5,739
- 33	- 46	- 17			- 43	-430		
84	33	142			18	158		

LESSON FORTY-TWO

It's time for another abacus lesson. As
usual let's begin this lesson by working a
problem in which you can use what you've
1 learned earlier. Get ready to subtract. OBV
90 - 5 - 16 - 2 - 8 - 12 - 19 - 4 - 3 - 4
- 5 - 3 - 9 + 10 - 2 Your final answer
should be 8.

Clear your abacus.

This time I'm going to read three phrases which describe a way of subtracting on the abacus. To show that you know what number can be subtracted with each rule set that number on your abacus.

2 Set 6, clear 1 left 4
Set 4, clear 1 left 6
Set 5, clear 1 left 5

Good. or Restatement of rule indicating its application.

Set 14 on your abacus. Is it possible to subtract 6 from that number by setting 4 and clearing 1 left?

3 NO

Because it is not possible to subtract 6 in that way, the number 14 should still be recorded on your abacus.

Is it possible to subtract 6 from 14 by clearing 1 and then clearing 5?

4 There is one other way of subtracting 6 which NO
we will learn in this lesson. The number 14
should still be recorded on your abacus.

If you subtracted 1 and then 5 you would have subtracted 6. Subtract 1 from 14. Then use the rule for subtracting 5 to help you subtract 5.

5 OBV

If you have subtracted correctly the number 8 should be recorded on your abacus. You may clear your abacus.

Try this simple problem. Set 13 on your abacus. Now subtract 6. First subtract 1, then use the rule that will help you subtract 5. The number 7 should be recorded on your abacus.

6 OBV

We may summarize what you did to subtract 6 by saying that you cleared 1, set 5 and cleared 1 left. Clear your abacus.

Here's another problem in which you'll need to subtract 6 by clearing 1, setting 5 and clearing 1 left. Set 43 on your abacus. From that number subtract 1 ten and 6 or 16.

7

$$\begin{array}{r} 43 \\ -16 \\ \hline 27 \end{array}$$

The difference is 27. Did you subtract correctly?

Clear your abacus.

YES

You'll also need the rule clear 1, set 5, clear 1 left in this problem. Set 131 on your abacus. From that number subtract 6 tens and 6 or 66. Look above and below the bar when you read your answer.

8

$$\begin{array}{r} 131 \\ -\ 66 \\ \hline 65 \end{array}$$

I agree. or Sixty-five is the correct answer.

Clear your abacus.

9

Set 222 on your abacus. From that number subtract 166. Look for a chance to use the rule clear 1, set 5, clear 1 left to subtract 6. Read me your answer.

Good. or The correct answer is 56.

Clear your abacus.

222
-166
56

10

I have given you a sheet which has three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used the rule clear 1, set 5, clear 1 left to subtract 6.

*139 - 65 = 74
* 91 - 36 = 55
98 - 66 = 32

You should have circled the first two problems.

ANSWERS STARRED

11

In this lesson you've learned to subtract 6 by clearing 1, setting 5 and clearing 1 left. What is the other rule that will help you subtract 6 on the abacus?

Good. or To set 4 and clear 1 left is a way of subtracting 6.

SET 4, CLEAR 1 LEFT

12

To work this problem you'll need to use both ways of subtracting 6. Set 91 on your abacus. Subtract 16. Now, subtract another 16.

Your final answer should be 59. Did you subtract accurately?

Clear your abacus.

91
-16
75
-16
59

YES

---

13 Set 38 on your abacus. Subtract 6 so that your abacus will say 32. Continue to subtract by 6's as long as it is possible. OBV

Good. or Your final answer should be 2.

---

14 This problem has large numbers in it. Set 12,953 on your abacus. From that number subtract 6,562.

$$\begin{array}{r} 12{,}953 \\ -\ 6{,}562 \\ \hline 6{,}391 \end{array}$$

Your final answer should be 6,391. Did you subtract accurately?

Clear your abacus. YES

Criterion Check

I'm going to read seven problems. Raise your hand if you would like me to repeat a problem. I'll ask you to read your answer aloud.

143	41	872	11,780	143	91	963
- 16	-26	-226	- 6,256	- 61	-56	-366
127	15	646	5,524	82	35	597
- 65	- 6			- 56	- 6	
62	9			26	29	

I'm going to read three phrases which explain how to subtract on the abacus. To show that you know what number can be subtracted with each rule set that number on your abacus.

15

Set 9, clear 1 left	1
Set 2, clear 1 left	8
Set 1, clear 1 left	9

Good. or Restatement of rule indicating its application.

16 This problem has several numbers in it. You may need to use a rule for subtracting 1 to help you when it is time to clear 1 left. Begin by setting 738 on your abacus. From that number subtract 467. Now subtract 79.

$$\begin{array}{r} 738 \\ -467 \\ \hline 271 \\ -\ 79 \\ \hline 192 \end{array}$$

Your final answer should be 192. Did you subtract accurately?

Clear your abacus. YES

---

17 There are three-digit numbers in this problem. Begin by setting 622 on your abacus. From that number subtract 246.

$$\begin{array}{r} 622 \\ -246 \\ \hline 376 \end{array}$$

Your final answer should be 376. Did you subtract accurately?

Clear your abacus. YES

---

18 Try this problem which has four-digit numbers in it. Begin by setting 7,144 on your abacus. From that number subtract 4 thousands, 6 hundreds, 6 tens and 1. Look above and below the bar when you read your answer.

$$\begin{array}{r} 7144 \\ -4661 \\ \hline 2483 \end{array}$$

I agree. or The correct answer is 2,483.

Clear your abacus.

Criterion Check

I'm going to read eight problems. I'll ask you to read your results aloud. Raise your hand if you would like me to repeat the problem.

115	501	954	11,626	129	1122	1531	19,220
- 24	-166	-276	- 3,561	- 63	- 465	- 864	-12,641
91	335	678	8,065	66	657	667	6,579
- 26	-142	- 89		- 27		- 589	
65	193	589		39		78	

LESSON FORTY-THREE

1 As usual let's begin this lesson by working a problem in which you can use what you've learned about subtraction. 122 - 62 - 1 - 27 - 5 - 14 - 3 - 6 + 41 - 3 - 13 - 3 - 2 - 4 - 4 - 1 - 1 - 4 - 2 Your final answer should be 8. OBV

Clear your abacus.

2 Set 14 on your abacus. Can you show that 14 - 7 = 7 by clearing 2 and then clearing 5? NO

There is another way of subtracting 7 which we will learn in this lesson. The number 14 should still be recorded on your abacus.

3 Let's try again to subtract 7 from 14. First subtract 2. Now use the rule set 5, clear 1 left to help you subtract 5. If that is a good way of subtracting 7, the number 7 should be recorded on your abacus. Is it?

Clear your abacus. YES

4 Let's try one more simple problem in which you can use that way of subtracting 7. Set 13 on your abacus. Now subtract the 2 and 5 of 7. You will need the rule for subtracting 5 to help you. You know that 13 - 7 = 6. Is the number 6 recorded on your abacus?

We can summarize what you did to subtract 7 by saying that you cleared 2, set 5 and cleared 1 left. You may clear your abacus. YES

5

Let's work another problem in which you can use the rule clear 2, set 5, clear 1 left to subtract 7. Set 43 on your abacus. From that number subtract 27.

$$\begin{array}{r} 43 \\ -27 \\ \hline 16 \end{array}$$

Your final answer should be 16. Did you subtract accurately?

Clear your abacus.

YES

6

Here's another problem in which you can use the rule clear 2, set 5, clear 1 left to subtract 7. Begin by setting 146 on your abacus. Now, subtract 7 tens and 5 or 75. Read me your answer.

$$\begin{array}{r} 146 \\ -\ 75 \\ \hline 71 \end{array}$$

Good. or The correct answer is 71.

Clear your abacus.

7

I have given you a sheet with three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used the rule clear 2, set 5, clear 1 left to subtract 7.

ANSWERS STARRED

*72 - 17 = 55
896 - 771 = 125
*139 - 73 = 66

You should have circled the first and third problems.

8 There is more than one way to subtract 7 on an abacus. In this lesson you have learned to use the rule clear 2, set 5, clear 1 left as a way of subtracting 7. I'm going to read three phrases which explain how to subtract on the abacus. When I have finished set 1, 2, or 3 on your abacus to show whether you think the first, second, or third phrase best explains a way of subtracting 7. ANSWER STARRED

Set 4, clear 1 left
*Set 3, clear 1 left
Clear 1, set 5, clear 1 left

To set 3 and clear 1 left is a way of subtracting 7.

---

9 To work this problem you'll need to use both ways of subtracting 7. Set 430 on your abacus. From that number subtract 277. Look above and below the bar when you read your answer.

$$\begin{array}{r} 430 \\ -277 \\ \hline 153 \end{array}$$

I agree. or The correct answer is 153.

Clear your abacus.

---

10 This time let's work a problem which has several numbers in it. To begin set 147 on your abacus. From that number subtract 76. Finally, subtract 17.

$$\begin{array}{r} 147 \\ -\ 76 \\ \hline 71 \\ -\ 17 \\ \hline 54 \end{array}$$

Your final answer should be 54. Did you subtract accurately?

Clear your abacus. YES

---

11 This time begin by setting 36 on your abacus. Now subtract 7 so that your abacus will say 29. Continue to subtract 7's until some number smaller than 7 appears on your abacus. OBV

The number 1 should be recorded on your abacus. You may clear your abacus.

Criterion Check

I'm going to read seven problems which you should be able to work independently. After you've finished, I'll ask one of you to read your answer aloud. If you'd like me to repeat a problem raise your hand.

83	138	222	18,673	134	149	912
-37	- 76	-177	-12,717	- 37	- 75	-777
46	62	45	5,956	97	74	135
	- 57				- 17	
	5				57	

12

Try this problem which has several numbers in it. Begin by setting 249 on your abacus. From that number subtract 6 tens and 7 or 67. Now subtract 97.

Your final answer should be 85. Did you subtract correctly?

Clear your abacus.

249
- 67
182
- 97
85

YES

13

Here's another problem similar to the last one. Begin by setting 154 on your abacus. First, subtract 57. Then, subtract 28. Read me your answer.

I agree. or The correct answer is 69.

Clear your abacus.

154
- 57
97
- 28
69

14

This problem has three-digit numbers in it. Think carefully as you work. Begin by setting 914 on your abacus. From that number subtract 137. Now subtract 281.

Your final answer should be 496. Did you subtract accurately?

Clear your abacus.

914
-137
777
-281
496

YES

15 The numbers in this problem have four- and five-digits. Begin by setting 12,210 on your abacus. From that number subtract 7,425. Look above and below the bar when you read your answer.

$$\begin{array}{r} 12{,}210 \\ -\ 7{,}425 \\ \hline 4{,}785 \end{array}$$

Good. or Your answer should be 4,785.

Clear your abacus.

Criterion Check

I'm going to read eight problems which you should be able to work. After you've finished, I'll ask one of you to read your answer aloud. Raise your hand if you would like me to repeat a problem.

123	920	953	12,410	133	421	522	10,258
- 37	-671	-297	- 4,672	- 74	-272	-167	- 1,721
86	249	656	7,738	59	149	355	8,537
- 27	- 66	-234		- 47	- 62	-196	
59	183	422		12	87	159	

LESSON FORTY-FOUR

1 Begin this lesson by setting 162 on your abacus. With that number as a start we'll work a subtraction problem in which you can use what you've already learned. The problem is 162 - 3 - 34 - 16 - 73 - 4 - 7 - 2 - 6 - 8 - 4 - 4 + 42 - 5 - 9 - 4 - 1 - 3 - 2 - 9 - 1 OBV
Your final answer should be 9.

Clear your abacus.

2 I'm going to read three phrases which explain a way of subtracting on the abacus. When I have finished, set a 1, 2, or 3 on your abacus to show whether you think the first, second, or third phrase best explains a way of subtracting 8. ANSWER STARRED

Set 5, clear 1 left
*Clear 3, clear 5
Set 3, clear 1 left

It is possible to subtract 8 by clearing 3 and then clearing 5.

3 Set 14 on your abacus. Is it possible to subtract 8 from 14 by clearing the 3 and 5 of 8? NO

The number 14 should still be recorded on your abacus.

4 Let's see if we can find another way of subtracting 8. First clear 3. Then use the rule for subtracting 5 to help you clear 5. The number 6 should be recorded on your abacus. Is it?

You may clear your abacus. YES

We can summarize what you did to subtract 8 by saying that you cleared 3, set 5 and cleared 1 left. Try it in this problem. Set 23 on your abacus. Then subtract 8.

$$\begin{array}{r}23\\-\ 8\\ \hline 15\end{array}$$

5

The number 15 should be recorded on your abacus. Did you subtract correctly?

Clear your abacus.

YES

This problem will give you more than one opportunity to use the rule clear 3, set 5, clear 1 left to subtract 8. Set 143 on your abacus. Now subtract 8 tens and 8 or

6 88. Read me your answer.

$$\begin{array}{r}143\\-\ 88\\ \hline 55\end{array}$$

I agree. or The correct answer is 55.

Clear your abacus.

YES

I have given you a sheet with three simple problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you subtracted 8 by clearing 3, setting 5, and clearing 1 left.

7

ANSWER STARRED

11 - 8 = 3
*13 - 8 = 5
9 - 8 = 1

You should have circled the second problem.

This time let's work a problem that has several numbers in it. To begin set 148 on your abacus. From that number subtract 85. Now, subtract 8.

$$\begin{array}{r}148\\-\ 85\\ \hline 63\\-\ \ 8\\ \hline 55\end{array}$$

8

Your final answer should be 55. Did you subtract accurately?

Clear your abacus.

YES

---

9 It is possible to use any rule of abacus calculation in either the ones, tens, hundreds, or thousands column. In this problem you will need to apply the rule clear 3, set 5, clear 1 left to subtract 8 in the ones column and in the hundreds column. Set 2,394 on your abacus. From that number subtract 1,838. Read me your answer.

$$\begin{array}{r} 2394 \\ -1838 \\ \hline 556 \end{array}$$

I agree. or The correct answer is 556.

Clear your abacus.

---

10 In this lesson you've learned to subtract 8 by clearing 3, setting 5 and clearing 1 left. What is the other rule for subtracting 8?

SET 2,
CLEAR 1
LEFT

Good. or To set 2 and clear 1 left is a way of subtracting 8.

---

11 To work this problem you'll need to use both ways of subtracting 8. Set 1,593 on your abacus. From that number subtract 888. Look above and below the bar when you read your answer.

$$\begin{array}{r} 1593 \\ -\ 888 \\ \hline 705 \end{array}$$

Good. or The correct answer is 705.

Clear your abacus.

Criterion Check

I'm going to read seven problems. I'll ask you to read your answer aloud and will repeat a problem if necessary.

$$\begin{array}{rrrrrrr} 93 & 1423 & 144 & 14,845 & 43 & 1343 & 143 \\ -68 & -\ 828 & -\ 82 & -\ 8,280 & -28 & -\ 882 & -\ \ 18 \\ \hline 25 & 595 & 62 & 6,565 & 15 & 461 & 125 \\ -18 & & -\ 58 & & -\ 8 & & -\ 18 \\ \hline 7 & & 4 & & 7 & & 107 \end{array}$$

12

I'm going to read four statements. If what I say is true raise your hand. If what I say is not true sit quietly.

To set 5 and clear 1 left is a way of subtracting 5.

Because 1 + 5 = 6, you can subtract 6 by clearing 1 and using the rule for subtracting 5.

Because 2 + 5 = 7, you can subtract 7 by clearing 2 and using the rule for subtracting 5.

Because 3 + 5 = 8, you can subtract by clearing 3 and using the rule for subtracting 5.

All of the statements were true.

13

Now I'm going to give you a sheet that has three phrases on it. To the right of each phrase are some numbers. Read each phrase and then circle the one number that can be subtracted using each rule.

ANSWER STARRED

Clear 2, set 5, clear 1 left 6 7* 8
Clear 1, set 5, clear 1 left 6* 7 8
Clear 3, set 5, clear 1 left 6 7 8*

Restatement of all rules indicating what number should have been marked.

14

There are three-digit numbers in this problem. Set 444 on your abacus. From that number subtract 286. Look above and below the bar when you read your answer.

$$\begin{array}{r} 444 \\ -286 \\ \hline 158 \end{array}$$

I agree. or The correct answer is 158.

Clear your abacus.

15	This problem also has three-digit numbers in it. Begin by setting 592 on your abacus. From that number subtract 357. Now subtract 81. Your final answer should be 154. Did you subtract accurately? Clear your abacus.	592 -357 235 - 81 154 YES
16	There are four- and five-digit numbers in this problem. Begin by setting 14,191 on your abacus. From that number subtract 8,486. Your final answer should be 5,705. Did you subtract accurately? Clear your abacus	14,191 - 8,486 5,705 YES
17	The numbers in this problem have three and four digits. Begin by setting 4,274 on your abacus. From that number subtract 3,368. Now subtract 213. Your final answer should be 693. Did you subtract accurately? Clear your abacus.	4274 -3368 906 - 213 693 YES
18	I'm going to read four rules of subtraction. To show that you know what number can be subtracted with each rule set that number on your abacus. Set 9, clear 1 left 1 Set 6, clear 1 left 4 Set 1, clear 1 left 9 Set 4, clear 1 left 6 Good. or Restatement of rule indicating its application.	

19 You will need to use all the rules we just talked about to work this problem. 120 - 6 - 4 - 1 - 9 - 90 - 4 OBV

The final answer should be 6.

Clear your abacus.

Criterion Check

I'm going to read eight problems which you should be able to work independently. After you've finished, I'll ask you to read your answer aloud. If you'd like me to repeat a problem, raise your hand.

32	2736	404	1431	141	9333	313	1359
-17	- 187	-218	- 856	- 43	-5478	-128	- 817
15	2549	186	575	98	3855	185	542
- 8		- 43	- 548	- 9		- 96	- 316
7		143	27	89		89	226

LESSON FORTY-FIVE

This is your last abacus lesson. In it you'll
learn a way of subtracting 9 on the abacus.
Before we begin something new let's, as usual,
work a problem in which you can use what you've
already learned. 165 - 9 - 15 - 6 - 8 - 3 - 8
1 - 42 - 7 - 5 - 4

Your final answer should be 58. Did you subtract accurately.

Clear your abacus. YES

Set 14 on your abacus so that we can talk
about how to subtract 9 from it. To clear
4 and clear 5 is one way of subtracting 9.
2 Can you do that in this case? NO

The number 14 should still be recorded on your abacus.

You have used the rule for subtracting 5 to
help you subtract the 1 and 5 of 6, the 2
and 5 of 7 and the 3 and 5 of 8. That way
3 of thinking may help you subtract the 4 and
5 of 9. Try it.

Since 14 - 9 = 5 the number 5 should be
recorded on your abacus. Is it? YES

We can summarize what you did to subtract 9
by saying that you cleared 4, set 5, and
4 cleared 1 left. Try it in this problem.
Set 34 on your abacus. From that number
subtract 1 ten and 9 or 19. The correct
answer is 15.

$$\begin{array}{r}34\\-19\\\hline 15\end{array}$$

Clear your abacus. OBV

5 Here's another problem in which you can use the rule clear 4, set 5, clear 1 left to subtract 9. Set 148 on your abacus. From that number subtract 95. Read me your answer.

```
 148
- 95
  53
```

I agree. or The correct answer is 53.

Clear your abacus.

---

6 I have given you a sheet with three problems on it. The correct answers have been given. Work each problem for yourself and then circle the problem or problems in which you used the rule clear 4, set 5, clear 1 left to subtract 9.

ANSWERS STARRED

*146 - 91 = 55
* 94 - 49 = 45
100 - 99 = 1

You should have circled the first two problems.

---

7 The rules of abacus calculation will apply in any column on the abacus. As you work this problem think whether you are using the rule clear 4, set 5, clear 1 left to subtract 9 ones, 9 tens, 9 hundreds, or 9 thousands. Set 1,479 on your abacus. From that number subtract 923. Your final answer should be 556. Did you apply the rule in the ones, tens, hundreds, or thousands column?

HUNDREDS

Good. or You should have applied that rule in the hundreds column.

Clear your abacus.

In this lesson you've learned to subtract 9 by clearing 4 and using the rule for subtracting 5, but there is more than one way of subtracting 9. I'm going to read three phrases which explain how to move beads on the abacus. Raise your hand each time you hear a phrase which explains a way of subtracting 9.

8 *Clear 4, set 5, clear 1 left — ANSWERS STARRED
*Clear 4, clear 5
*Set 1, clear 1 left

It is possible to subtract 9 in all three ways.

To work this problem you'll need to use more than one way of subtracting 9. Begin by setting 1,849 on your abacus. From that number subtract 999. Read me your answer.

9

Good. or The correct answer is 850.

Clear your abacus.

1849
- 999
850

---

10 This time begin by setting 84 on your abacus. Subtract 9 from that number so that your abacus will say 75. Continue to subtract by 9's until either 28, 29, or 30 is recorded on your abacus. OBV

Thirty is the correct answer. You may clear your abacus.

Criterion Check

I'm going to read seven problems which you should be able to work independently. After you've both finished, I'll ask one of you to read your answer aloud. If you'd like me to repeat a problem raise your hand.

149	115	144	248	164	242	1496
- 99	- 19	- 19	- 93	- 19	- 91	- 959
50	96	125	155	145	151	537
		- 19			- 91	
		106			60	

11	Let's work a problem which has several numbers in it. Begin by setting 149 on your abacus. From that number subtract 96. Now subtract 8. Your final answer should be 45. Did you subtract accurately? Clear your abacus.	149 - 96 53 - 8 45 YES
12	Here's another problem similar to the last one. Work carefully. Set 102 on your abacus. Subtract 7 from that number. Now subtract 69. Read me your answer. Good. or The correct answer is 26. Clear your abacus.	102 - 7 95 - 69 26
13	There are three- and four-digit numbers in this problem. Begin by setting 4,444 on your abacus. From that number subtract 962. Your final answer should be 3,482. Did you subtract accurately? Clear your abacus.	4444 - 962 3482 YES
14	Try this problem. Begin by setting 3,460 on your abacus. From that number subtract 654. Look above and below the bar when you read your answer. I agree. or Your answer should be 2,806. Clear your abacus.	3460 - 654 2806
15	You've learned that the rule for 5 may help you to subtract 6, 7, 8, or 9. You'll need that idea to work this problem. 294 - 69 - 61 - 7 - 14 - 8 Your final answer should be 135. Did you subtract correctly? Clear your abacus.	YES

This problem will give you an opportunity to use almost everything you know about subtracting on the abacus. It's a long problem, but I'll read slowly so that you'll have plenty of time to work accurately.
16 94 - 9 - 8 - 9 - 45 - 8 - 7 - 8 + 41 - 6 - 6 - 4 - 2 OBV

Your final answer should be 23. You may clear your abacus.

Criterion Check

I'm going to read eight problems which you should be able to work independently. After you've finished I'll ask one of you to read your answer aloud. If you'd like me to repeat a problem raise your hand.

842	4342	934	1424	934	3490	1424	1248
-289	-4167	-679	- 908	-789	-2924	- 719	- 486
553	175	255	516	145	566	705	762
		-127	- 127			- 13	- 157
		128	389			692	605
							- 35
							570

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